ECON-UN 3211 - Intermediate Microeconomics

Recitation 10: Final Review

Matthew Alampay Davis

December 9, 2022

Pre-exam resources

1. Isaac's Extra Review problems (+ solutions)
2. I'm preparing notes running through the course topics at a conceptual level
3. My recitation recordings and slides (including feedback on midterm)
4. Feedback on problem set 6 (even if you got a 10)
5. Varian textbook in my folder (very good on imperfect competition)
6. Will update my Running Notes at some point; send me any specific questions
7. No promises but possible Zoom office hours the week of the exam

Plan for today

Bertrand competition: two cases

Equilibrium under Cournot vs. Stackelberg competition

Competitive equilibrium (Recitation 7, Practice Problem 2)

Any other topics to revisit? Easy to pull up slides or practice problems to go over

Bertrand competition: two cases

From Recitation 9, practice problem 3:

a) What price would each duopolist set if the other duopolist didn't exist

$p^D(Q) = 2400 - Q \Rightarrow TR(Q) = p^D(Q) \cdot Q$

$= (2400 - Q)Q$

$= 2400Q - Q^2$

$\Rightarrow MR(Q) = 2400 - 2Q$

- Market demand

$$Q^D(p) = 2400 - p$$

Firm 1: $MR = MC_1$ when $2400 - 2Q_1^M = 20$

$\Rightarrow Q_1^M = \frac{2400-20}{2} = 1190$

$\Rightarrow p_1^M = p^D(1190)$

$= 2400 - 1190 = 1210$

- Production costs

$$c_1(Q) = 20Q \quad MC_1 = 20$$

$$c_2(Q) = 10Q \quad MC_2 = 10$$

Firm 2: $MR = MC_2$ when $2400 - 2Q_2^M = 10$

$\Rightarrow Q_2^M = \frac{2400-10}{2} = 1195$

$\Rightarrow p_2^M = p^D(1195)$

$= 2400 - 1195 = 1205$

From Recitation 9, practice problem 3:
b) What is the outcome of Bertrand competition

- Market demand

$$Q^D(p) = 2400 - p$$

- Production costs

$$c_1(Q) = 20Q$$
$$c_2(Q) = 10Q$$

Suppose firm 1 begins as a monopolist

$\Rightarrow$ Sets $p = p_1^M = 1210$

Then firm 2 enters

$BR_2(p_1 = 1210) = 1205$ (just for this example, assume whole number prices)

Firm 1 $(p_2 = 1205) = 1204$

⋮

Firm 1 sets price at 21

$BR_2(p_1 = 21) = 20$

$BR_1(p_2 = 20) = 20$

$BR_2(p_1 = 20) = 19$ or $20 - \varepsilon$ (without the whole number assumption for arbitrarily small $\varepsilon > 0$)

$BR_1(20 - \varepsilon) = 20$

mutual best responses, $(p_1 = 20, p_2 = 20 - \varepsilon)$ is the Bertrand Nash equilibrium

From Problem Set 9, problem 2

b) If Firm A was a monopolist, what price would it charge?

- Market demand

$$Q^D(p) = 480 - 2P$$

- Production costs

$$c_A(Q) = 120Q$$
$$c_B(Q) = 240Q$$

If firm A was a monopolist

$p^D(Q) = 240 - \frac{Q}{2}$

$TR(Q) = p^D(Q) \cdot Q$

$= 240Q - \frac{Q^2}{2}$ $\Rightarrow$ $MR(Q) = 240 - Q$

$\Rightarrow$ Set $MR_A = MC_A$

$\Rightarrow 240 - Q = 120$

$\Rightarrow Q_A^M = 120$

$\Rightarrow p_A^M = 240 - \frac{120}{2}$

$= 180$

$Q_A^M = 120$, $p_A^M = 180$

From Problem Set 9, problem 2

c) Calculate the Nash Equilibrium (approximately if needed)

- Market demand

$$Q^D(p) = 480 - 2P$$

- Production costs

$$c_A(Q) = 120Q$$

$$c_B(Q) = 240Q$$

If A sets $P_A = 240$

$BR_B(P_A = 240) = 240$

why?

$BR_A(P_B = 240) = 180$ NOT 239 or 240-ε

$\pi_A(P_A) = P_A \cdot Q_A(P_A) - 120\, Q_A(P_A)$

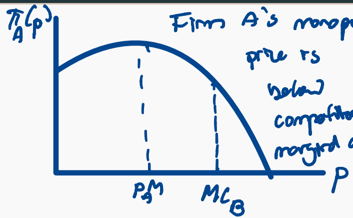
$= P_A(480 - 2P_A) - 120(480 - 2P_A)$

$= 480P_A - 2P_A^2 - 120 \cdot 480 - 240P_A$

$\frac{d\pi_A(P_A)}{dP_A} = 0 = 480 - 4P_A - 240$

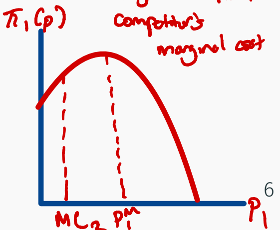
$\Rightarrow P_A^* = 180$

$Q_{NE} = 480 - 2(P_{NE}) = 120$



Firm A's monopoly price is below competitor's marginal cost

In first example, Firm 1's monopoly price was greater than competitor's marginal cost



From Problem Set 9, problem 2

d) Calculate the Deadweight Loss of the Nash Equilibrium

- Market demand

$$Q^D(p) = 480 - 2P$$

- Production costs

$$c_A(Q) = 120Q$$
$$c_B(Q) = 240Q$$

The competitive equilibrium is when market demand meets the lowest cost function so that firms are supplying the good at cost most efficiently.

In this case, it's when firm A supplies the entire market at price $= MC_A = 120$

$$CE = P^*_{CE} = 120, \quad Q^*_{CE} = Q^D(P^*_{CE}) = 480 - 2(120) = 240$$

$$\updownarrow$$

$$NE = \{P_{NE} = 160, \; Q_{NE} = 120\}$$

From Problem Set 9, problem 2

d) Calculate the Deadweight Loss of the Nash Equilibrium

DWL

$$DWL = \frac{1}{2}(Q_{CE} - Q_{NE})(P_{CE} - P_{NE})$$
$$= \frac{1}{2}(240 - 120)(120 - 180)$$
$$= -\frac{1}{2}(120)(60)$$
$$= -3600$$

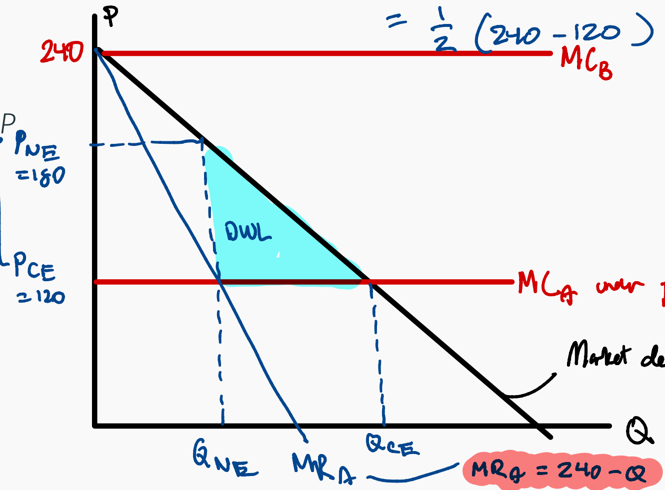
- Market demand

$$Q^D(p) = 480 - 2P$$

$P_{NE} - P_{CE} =$ monopolist markup

- Production costs

$$c_A(Q) = 120Q$$
$$c_B(Q) = 240Q$$



Discussion

- The setup of both games are very similar: two duopolists with different constant marginal costs facing a linear market demand function
- The key here is that profit functions are concave: even if you out-compete an opponent on prices, that does not mean you should set the highest possible price below their marginal cost
- In the first case, firm 1's monopolist profit function was maximized at quantity 1210, well above either firm's marginal costs of 20 and 10 so that meant profits were decreasing in price
- But in the second case, firm A's monopolist profit function was maximized at price 180, above its marginal cost but also well below firm B's marginal cost
- Deadweight loss is calculated relative to the case where the market demand meets the most efficient supply

Equilibrium under Cournot vs. Stackelberg competition

From Final Review Problem 13

a) Calculate the Nash Equilibrium if these duopolists were to enter Cournot competition

- Market demand

$$Q^D(p) = 360 - p$$

- Production costs

$$c_1(q) = 12q$$
$$c_2(q) = 24q$$

$p^D(Q) = 360 - Q$, $Q = q_1 + q_2$ (market supply)

$p^D(q_1, q_2) = 360 - q_1 - q_2$

$\Rightarrow \pi_2(q_2) = p^D(q_2)q_2 - 24q_2$

$= (360 - q_1 - q_2)q_2 - 24q_2$

$= (336 - q_1)q_2 - q_2^2$

$\pi_1(q_1) = p^D(q_1)q_1 - 12q_1$

$= (360 - q_1 - q_2)q_1 - 12q_1$

$= (348 - q_2)q_1 - q_1^2$

$FOC_2: 336 - q_1 = 2q_2$

$FOC_1: 348 - q_2 = 2q_1$

$FOC_2 - FOC_1$:

$-12 - q_1 + q_2 = 2(q_2 - q_1)$

$\Rightarrow \boxed{q_2 = q_1 - 12}$

From Final Review Problem 13

a) Calculate the Nash Equilibrium if these duopolists were to enter Cournot competition

- Market demand

$$Q^D(p) = 360 - p$$

- Production costs

$$c_1(q) = 12q$$
$$c_2(q) = 24q$$

$q_2 = q_1 - 12$

Plug into either FOC:

$336 - q_1 = 2q_2$

$= 2(q_1 - 12)$

$\Rightarrow 336 - q_1 = 2q_1 - 24$

$\Rightarrow 360 = 3q_1$

$\Rightarrow q_1 = 120$

$\Rightarrow q_2 = 108$

$\Rightarrow Q = q_1 + q_2 = 228$

$\Rightarrow p(Q) = 360 - 228 = 132$

Nash Equilibrium is given by

1. $q_1^* = 120$
2. $q_2^* = 108$
3. $Q^* = 228$
4. $p^* = 132$

Need all four values for a valid equilibrium!

From Final Review Problem 13

a) Calculate the Subgame Perfect Nash Equilibrium if these duopolists were to enter Stackelberg competition with firm 1 playing first

- Market demand

$$Q^D(p) = 360 - p$$

- Production costs

$$c_1(q) = 12q$$
$$c_2(q) = 24q$$

Similar but there is a timing element.

- Firm 1 chooses q_1 first
- Firm 2 best responds to q_1 : $q_2(q_1)$ the best response function

Key here: because of timing, q_1 is definite : fully credible move that firm 2 has to respond to.

In Cournot, lack of credibility in simultaneous game leads to a different Nash Equilibrium.

To solve, work backwards:

- firm 2 best responds to any choice q_1 by Firm 1
- firm 1 anticipates this best response and chooses q_1 that leads to the best outcome for them : $U_1(q_1, q_2(q_1))$

From Final Review Problem 13

a) Calculate the Subgame Perfect Nash Equilibrium if these duopolists were to enter Stackelberg competition with firm 1 playing first

- Market demand

$$Q^D(p) = 360 - p$$

- Production costs

$$c_1(q) = 12q$$
$$c_2(q) = 24q$$

$$\pi_2(q_2 | q_1) = (360 - q_1 - q_2)q_2 - 24q_2$$
$$= (360 - q_1 - 24)q_2 - q_2^2$$

$$FOC_2 : 0 = 336 - q_1 - 2q_2$$
$$\Rightarrow q_2(q_1) = \frac{336 - q_1}{2}$$, firm 2's best response function

$$\pi_1(q_1 | q_2(q_1)) = (360 - q_1 - q_2(q_1))q_1 - 12q_1$$
$$= \left(348 - q_1 - \frac{336 - q_1}{2}\right)q_1$$
$$= (348 - 168)q_1 - q_1^2 + \frac{q_1^2}{2}$$

$$FOC_1 : 0 = 180 - 2q_1 + q_1 \Rightarrow q_1^* = 180$$

$$\Rightarrow q_2^* = q_2(q_1^*) = \frac{336 - 180}{2} = 78 \quad \Rightarrow Q^* = q_1^* + q_2^* = 258$$

From Final Review Problem 13

a) Calculate the Subgame Perfect Nash Equilibrium if these duopolists were to enter Stackelberg competition with firm 1 playing first

$$p^* = 360 - Q^*$$
$$= 360 - 258$$
$$= 102$$

- Market demand

$$Q^D(p) = 360 - p$$

$\Rightarrow$ the SPNE is given by

- Production costs

$$\{p^* = 102,\ Q^* = 258,\ q_1^* = 180,\ q_2^* = 78\}$$

$$c_1(q) = 12q$$

$$c_2(q) = 24q$$

Compare to Cournot NE:

$$\{p^* = 132,\ Q^* = 228,\ q_1^* = 120,\ q_2^* = 108\}$$

SPNE where more efficient firm moves first: lower price, higher quantity, $q_1 \uparrow$, $q_2 \downarrow$

Discussion

- In comparing Bertrand and Cournot, we said that the first was more cut-throat: unless firms are identical, the one with the lower marginal cost might relinquish profit but still dominate the entire market. Cournot is more accommodating as it allows firms of different efficiency to co-exist.
- We also said that their relevance depends on context: Bertrand better describes markets where firms can mobilize production very quickly, enter the market very easily, and consumers are very responsive while Cournot better describes markets where firms simultaneously make binding decisions ahead of time
- In Cournot vs. Stackelberg, the setups are very similar but the introduction of a timing element in Stackelberg competition gives a significant edge to the firm that chooses first
- Stackelberg thus better applies to markets where there is a clear first-mover advantage

Competitive equilibrium (Recitation 7, Practice Problem 2)

Competitive equilibrium (Recitation 7, Practice Problem 2)

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(a) What is the aggregate demand in this market?

Competitive equilibrium (Recitation 7, Practice Problem 2)

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(b) What is the aggregate supply in this market?

Competitive equilibrium (Recitation 7, Practice Problem 2)

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(c) Find the competitive equilibrium

Competitive equilibrium (Recitation 7, Practice Problem 2)

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(d) What is consumer/producer surplus?

Competitive equilibrium (Recitation 7, Practice Problem 2)

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are ~~50~~ **10** identical firms

(e) What is the new equilibrium?

Competitive equilibrium (Recitation 7, Practice Problem 2)

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are ~~50~~ **10** identical firms

(f) What is the new consumer/producer surplus?

Competitive equilibrium (Recitation 7, Practice Problem 2)

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are ~~50~~ **10** identical firms

(g) Do these changes in surplus make sense?

- The consumer population has not changed but prices have increased
- Thus, fewer consumers are being served so we should expect a decrease in CS
- There are fewer firms but prices have increased so effect on producer surprlus ambiguous
- However, prices have increased more than the quantity has decreased so total producer surplus has increased
- Each individual firm experiences an even more significant increase in surplus

Any other topics to revisit? Easy to pull up slides or practice problems to go over