3. Exchange economies with an Edgeworth box
(b) Graph the contract curve, the set of Pareto improvements, and the core

Endowments w

1. $w^A = (75, 25)$
2. $w^B = (25, 25)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
2. Perfect substitutes: $U^B(x^B) = x_1^B + x_2^B$

Plug endowment point into the utility functions

$\bar{u}^A = 75 \cdot 25 = 1875$, $\bar{u}^B = 25 + 25 = 50$

This gives the equations for the indifference curves that intersect the endowment point:

$1875 = x_1^A x_2^A$ $\quad 50 = x_1^B + x_2^B$

$\Rightarrow x_2^A = \frac{1875}{x_1^A}$ $\quad \Rightarrow x_2^B = 50 - x_1^B$

Rewrite the second equation in terms of x_1^A and x_2^A

$\Rightarrow (50 - x_2^A) = 50 - (100 - x_1^A)$

$\Rightarrow x_2^A = 100 - x_1^A$

The indifference curves intersect where these are equal

$\frac{1875}{x_1^A} = 100 - x_1^A \Rightarrow x_1^A = \{25, 75\} \Rightarrow x_2^A = \{75, 25\}$

So they intersect at $(25, 75)$ and $(75, 25)$. Note $(25, 75)$ is outside the Edgeworth Box