ECON-UN 3211 - Intermediate Microeconomics

Recitation 8: Exchange economies

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November 18, 2022

Plan for today and, if necessary, beginning of next week

Problem Set 6 Feedback

Exchange economies

Practice question 1: Interior solutions

Pricing in exchange economies: market clearing and excess supply/demand

Practice question 2: Excess demand and market clearing

Practice question 3: Non-interior solutions in exchange economies

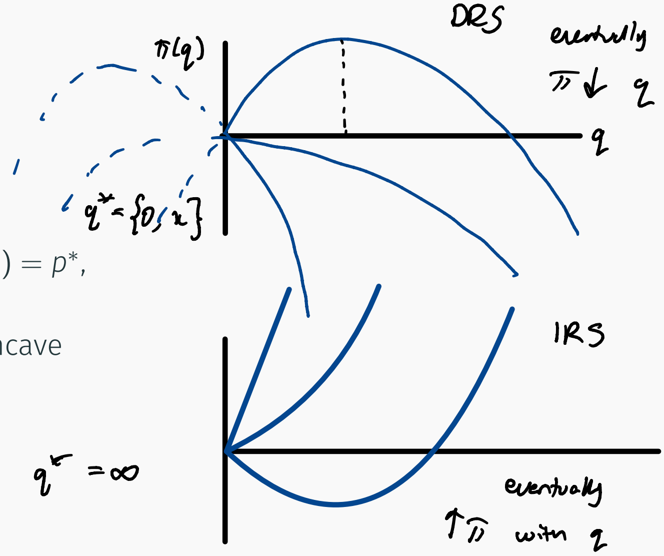
Problem Set 6 Feedback

Q1. Supply with Cobb-Douglas production $F(L, K) = L^{\alpha}K^{\beta}$

- Conditional factor demands
  - $L(q, w, r) = \gamma_L(w, r)q^{\frac{1}{\alpha+\beta}}$
  - $K(q, w, r) = \gamma_K(w, r)q^{\frac{1}{\alpha+\beta}}$
- $C(q, w, r) = \gamma_C(w, r)q^{\frac{1}{\alpha+\beta}}$
- $C''(q, w, r) < 0 \Leftrightarrow \alpha + \beta < 1$

Under perfect competition, $MR(q) = p^*$, a constant. Thus, strictly convex (concave) costs imply strictly concave (convex) profit:

1. DRS $\Leftrightarrow$ $\pi(q)$ strictly concave
2. IRS $\Leftrightarrow$ $\pi(q)$ strictly convex



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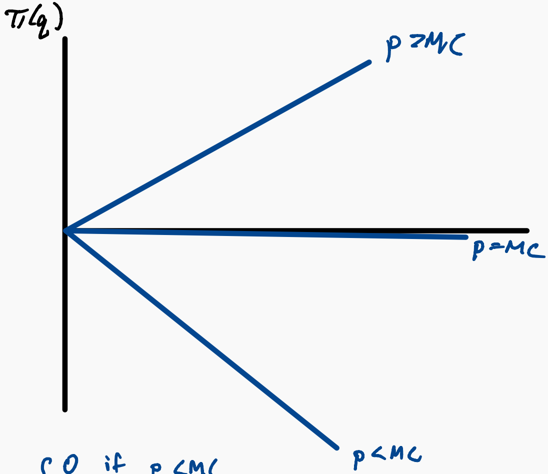
- $C''(q, w, r) < 0 \Leftrightarrow \alpha + \beta < 1$

Part e investigates the case where $\alpha + \beta = 1$. Here, the cost (and thus profit) functions are weakly convex and concave, i.e., linear. We find that the supply decision depends on p



$$q^{s*} = \begin{cases} 0 & \text{if } p < MC \\ [0,\infty] & p = MC \\ \infty & \text{if } p > MC \end{cases}$$

Q1. Supply with Cobb-Douglas production $F(L, K) = L^\alpha K^\beta$

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- $C(q, w, r) = \gamma_C(w, r)q^{\frac{1}{\alpha+\beta}}$
- $C''(q, w, r) < 0 \Leftrightarrow \alpha + \beta < 1$

Part f asks us to synthesize our findings: how does supply choice relate to $\alpha + \beta$, i.e., the returns to scale of production?

- From part c-d, we saw that DRS implies a finite quantity supplied, which may be zero
- Also from part c-d, we would infer that IRS implies infinite supply
- From part e, we saw that profits are linear but the slope depends on how $MR(q) = p^*$ compares to $MC(q)$
  - If $p < MC$, profits are increasing in q and $q^S = \infty$
  - If $p > MC$, profits are decreasing in q and $q^S = 0$
- What about when $p = MC$?

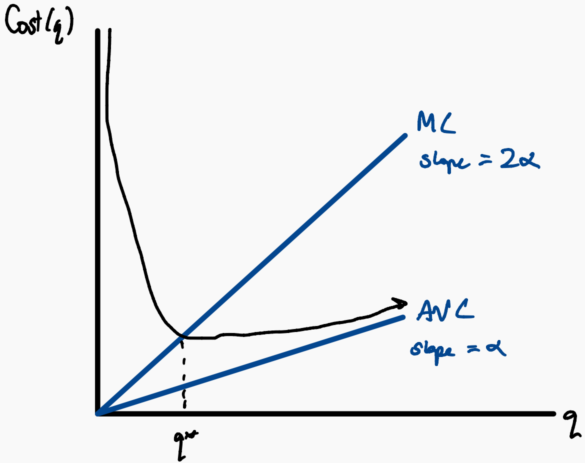
Q2.Production with $C(q) = \alpha q^2 + FC$

- $AC(q) = \alpha q + \frac{FC}{q}$
- $MC(q) = 2\alpha q$
- $VC(q) = \alpha q^2$
- $AVC(q) = \alpha q$

From parts a and b, $q^* = \sqrt{\frac{FC}{\alpha}}$ is where

1. $AC(q)$ is minimized
2. $MC(q)$ is equal to $AC(q)$

Part c asks us to draw a graph of MC, AC, and AVC. We should intuit that we want to include the results from a and b in that graph.



Exchange economies

General equilibrium theory

General equilibrium the method

- Exogenous primitives (number of agents, number of goods, technologies, preferences, endowments)
- Simultaneous mutual resolution of agents behaviors as specified by primitives
- Result: “equilibrium” prices and quantities

General equilibrium the theory

- Assumptions: perfect competition
  1. Price-taking behavior
  2. Symmetric information
  3. Markets for all goods
- These strong assumptions buy us an elegant theory integrating consumer and producer theory
- Key result: the fundamental theorems of welfare economics

Pure exchange: the Edgeworth box

Three basic economic activities:

1. Consumption
2. Production
3. Exchange

The Edgeworth box is a very simplified general equilibrium model of a *pure* exchange economy

- No production, just endowments $w = (w^A, w^B)$
- Given initial endowments, preferences over the goods determine what exchange unfolds

Very simple and abstract

- Two agents exchange two goods according to initial endowments and the interaction of their preferences
- No production, firms, or mediums of exchange (i.e., money)

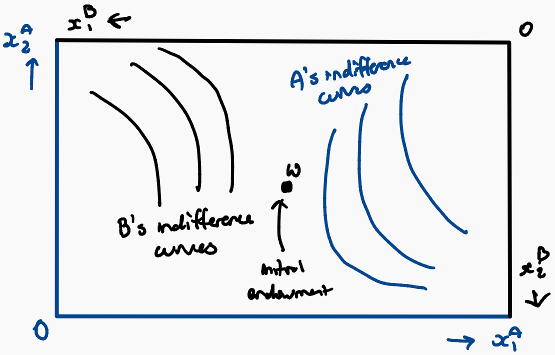
But rich in its insights

- Depicts how prices arise as a mechanism for clearing markets
- Depicts how preferences and endowments give rise to 'wealth'
- Depicts market efficiency and opportunities for mutual benefit

The Edgeworth box: capturing insights from consumer theory

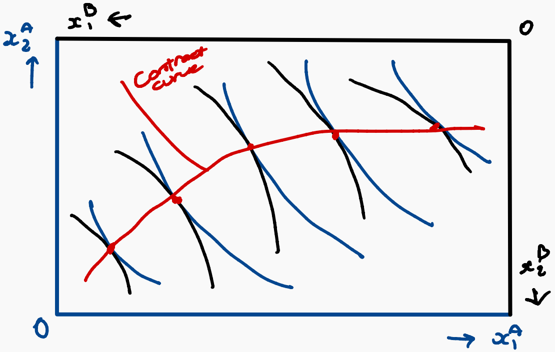
We can plot the familiar lines from consumer theory

- Wealth/income as the product $p \cdot q$
- Budget lines divide the two budget sets capture relative prices between two goods
- Indifference curves reflect the two consumers' preferences
- Tangency condition for well-behaved preferences depict optimal consumer behaviors
- Tracing these bundles as budget line changes gives us offer curves



The Edgeworth box: Pareto optimality and contract curves

- An allocation x in the Edgeworth box is **Pareto optimal/efficient** if there is no other feasible allocation x' that makes one consumer better off without making another worse off
- Trace out the points where indifference curves are just tangent to one another: the **contract curve**
- For given initial endowments, only a subset of the contract curve is a Pareto improvement: we call this the **core**



Pareto set given by $MRS^A = MRS^B$

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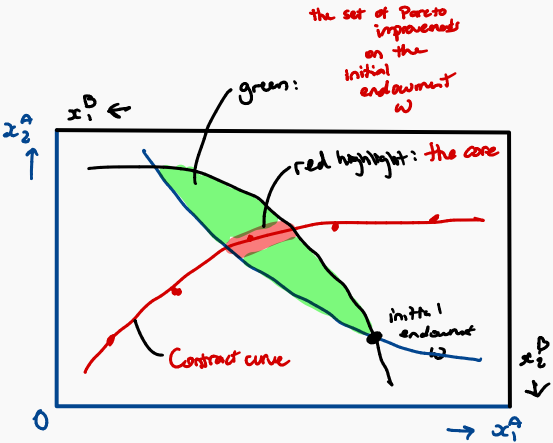


Practice question 1: Interior solutions

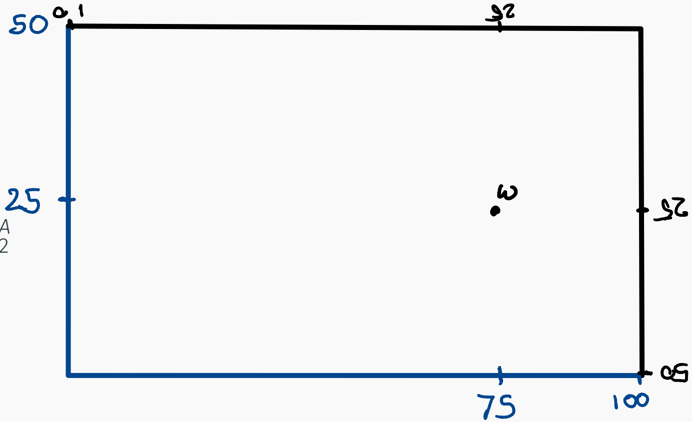
(a) Draw the Edgeworth box of this economy, depicting the initial endowment

Endowments w

1. $w^A = (75, 25)$
2. $w^B = (25, 25)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
2. Perfect substitutes: $U^B(x^B) = x_1^B + x_2^B$



Exchange economies with an Edgeworth box

(b) Graph the contract curve, the set of Pareto improvements, and the core

Endowments w

1. $w^A = (75, 25)$
2. $w^B = (25, 25)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
2. Perfect substitutes: $U^B(x^B) = x_1^B + x_2^B$

Plug endowment point into the utility functions

$\bar{u}^A = 75 \cdot 25 = 1875$, $\bar{u}^B = 25 + 25 = 50$

This gives the equations for the indifference curves that intersect the endowment point:

$1875 = x_1^A x_2^A$ $\qquad$ $50 = x_1^B + x_2^B$

$\Rightarrow x_2^A = \dfrac{1875}{x_1^A}$ $\qquad$ $\Rightarrow x_2^B = 50 - x_1^B$

Rewrite the second equation in terms of x_1^A and x_2^A

$\Rightarrow (50 - x_2^A) = 50 - (100 - x_1^A)$

$\Rightarrow x_2^A = 100 - x_1^A$

The indifference curves intersect when these are equal

$\dfrac{1875}{x_1^A} = 100 - x_1^A \Rightarrow x_1^A = \{25, 75\} \Rightarrow x_2^A = \{75, 25\}$

So they intersect at (25, 75) and (75, 25). Note (25, 75) is outside the Edgeworth Box

Exchange economies with an Edgeworth box
(b) Graph the contract curve, the set of Pareto improvements, and the core

Endowments w

1. $w^A = (75, 25)$
2. $w^B = (25, 25)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
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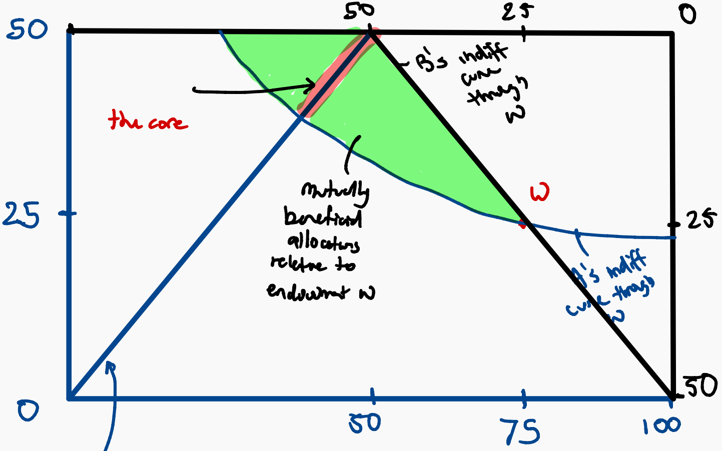


Contract curve: $MRS^A = MRS^B$

$\Rightarrow \dfrac{MU_1^A}{MU_2^A} = \dfrac{MU_1^B}{MU_2^B}$

$\Rightarrow x^B / x_1^A = 1/1 \Rightarrow x_1^B = x_1^A$

Exchange economies with an Edgeworth box
(c) Graph the contract curve, the set of Pareto improvements, and the core

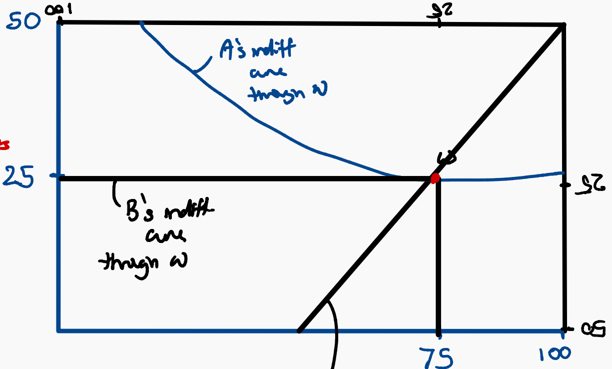
The set of Pareto improvements is empty : empty core
no Pareto improvements available

Endowments w

1. $w^A = (75, 25)$
2. $w^B = (25, 25)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
2. Perfect ~~substitutes~~ **complements**: $U^B(x^B) = \min\{x_1^B, x_2^B\}$



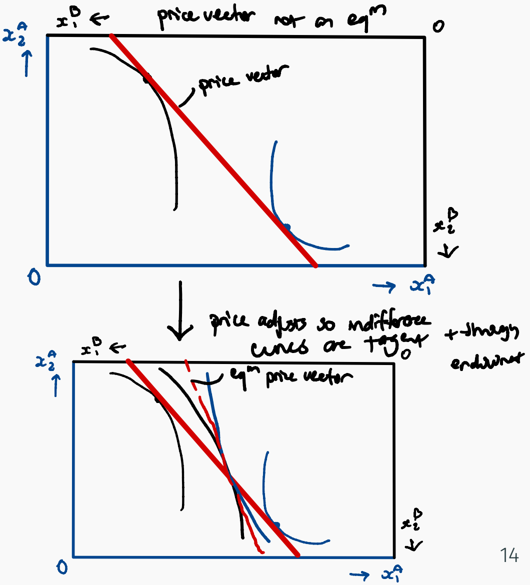
Contract curve : $MRS^A = \frac{MU_1^A}{MU_2^A} = \frac{x_2^A}{x_1^A}$

MRS^B = undefined : optimal where $x_2^B = x_1^A$ (complementarity ratio) ⇒ the Pareto set

Pricing in exchange economies: market clearing and excess supply/demand

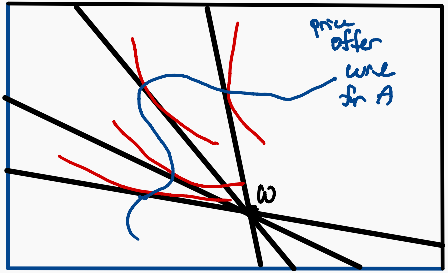
Excess supply and demand in exchange economies

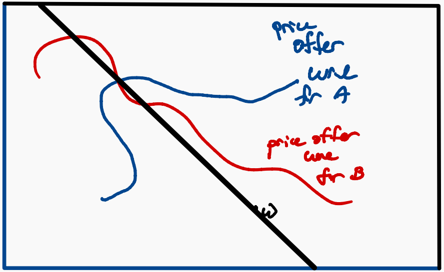
- Given a price vector, we see that the two tangent indifference curves may occur at different points
  - Consumer 1 is in net demand for good 2, consumer 2 in net supply
- Equilibrium adjustment: change the relative prices of the two goods so they are both tangent to the budget line at the same point
- In fact, there exists a unique budget line through the initial endowment that allows the market to clear



Approach 1: Using offer curves

- We can draw any number of budget lines through the endowment point, each corresponding to different tradeoffs between the two goods (i.e. relative prices)
- For each of these budget lines, we can trace out where the tangency condition is satisfied: this is the consumer's offer curve that passes through
- For well-behaved preferences, there is a unique budget line where the two offer curves intersect and the initial endowment ω





Approach 1: Using offer curves

- Mathematically, this is equivalent to setting total demands equal to total resources:

$$x_1^A(p, w^A) + x_1^B(p, w^B) = w_1$$
$$x_2^A(p, w^A) + x_2^B(p, w^B) = w_2$$

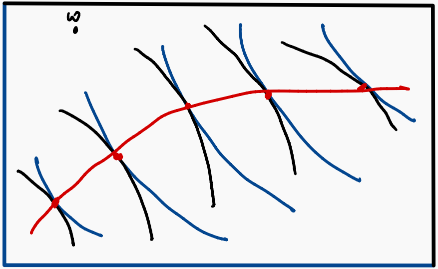
  and solving for the unique p that solves the system of equations

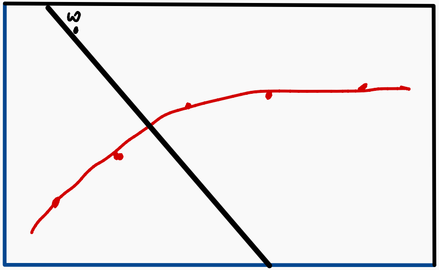
Approach 2: Using the contract curve

- Equivalently, we can find the budget line that satisfies both tangency conditions simultaneously:

$$MRS^A = \frac{p_1}{p_2} = MRS^B$$

- We already have an expression for all points where the two agents' MRS is identical: the contract curve
- Thus, there is a unique budget line through the initial endowment that clears the market and its slope is orthogonal/perpendicular to the contract curve





Practice question 2: Excess demand and market clearing

Market clearing in exchange economies

(a) Graph the contract curve for this economy

① Contract curve

$$MRS^A = \frac{x_2^A}{x_1^A} \qquad MRS^B = \frac{x_2^B}{x_1^B} = \frac{800 - x_2^A}{400 - x_1^A}$$

$$\Rightarrow (400 - x_1^A)\, x_2^A = (800 - x_2^A)\, x_1^A$$

$$\Rightarrow 400 x_2^A \underline{- x_1^A x_2^A} = 800 x_1^A \underline{- x_2^A x_1^A}$$

$$\Rightarrow x_2^A = 2 x_1^A$$

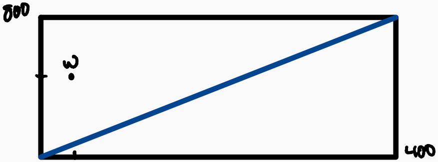
Endowments w

1. $w^A = (50, 500)$
2. $w^B = (350, 300)$

400 800

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
2. Cobb-Douglas: $u^B(x^B) = x_1^B x_2^B$



Market clearing in exchange economies
(b) Find Agent A's Marshallian demand for good 1

Symmetric Cobb-Douglas : spend 1/2 wealth on both goods

$$\Rightarrow w^A = p_1 \times 50 + p_2 \times 500$$

$$\Rightarrow x^A(p, w^A) = \left(\frac{50p_1 + 500p_2}{p_1} \times \frac{1}{2}, \frac{50p_1 + 500p_2}{2p_2} \right)$$

Endowments w

1. $w^A = (50, 500)$
2. $w^B = (350, 300)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
2. Cobb-Douglas: $u^B(x^B) = x_1^B x_2^B$

Market clearing in exchange economies

(c) Show how $p_1 = p_2 = 1$ fails to clear the market

Endowments w

1. $w^A = (50, 500)$
2. $w^B = (350, 300)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
2. Cobb-Douglas: $u^B(x^B) = x_1^B x_2^B$

Demands

1. $x^A(p, w^A) = \left(\frac{50p_1+500p_2}{2p_1}, \frac{50p_1+500p_2}{2p_2}\right)$

2. $x^B(p, w^B) = \left(\frac{350p_1+300p_2}{2p_1}, \frac{350+300p_2}{2p_2}\right)$

If $p_1 = p_2 = 1$

$$x_1^A(p, w^A) + x_1^B(p, w^B)$$

$$= \frac{50(1)+500(1)}{2(1)} + \frac{350(1)+300(1)}{2(1)}$$

$$= \frac{550+650}{2} = 600$$

$$> 400 = w^A + w^B$$

Market clearing in exchange economies
(d) Find a competitive equilbrium by Approach 1

Endowments w

1. $w^A = (50, 500)$
2. $w^B = (350, 300)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
2. Cobb-Douglas: $u^B(x^B) = x_1^B x_2^B$

Demands

1. $x_1^A(p, w^A) = \frac{50p_1+500p_2}{2p_1}$

$$x_1^A(p, w^A) + x_1^B(p, w^B) = \omega_1 = 400$$

$$\Rightarrow \frac{400p_1 + 800p_2}{2p_1} = 400$$

$$\Rightarrow 400p_1 + 800p_2 = 800p_1$$

$$\Rightarrow 800p_2 = 400p_1$$

$$\Rightarrow 2 = \frac{p_1}{p_2}$$

Competitive Equilibrium exists when $p^* = 2 = \frac{p_1}{p_2}$

(plug $\frac{p_1}{p_2} = 2$ into demand functions) $\Rightarrow$ $x^A(p, w^A) = (150, 300)$, $x^B = (250, 500)$

Market clearing in exchange economies
(e) Find a competitive equilbrium by Approach 2

Endowments w

1. $w^A = (50, 500)$
2. $w^B = (350, 300)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$
2. Cobb-Douglas: $u^B(x^B) = x_1^B x_2^B$

Demands

1. $x_1^A(p, w^A) = \frac{50p_1+500p_2}{2p_1}$

Contract curve equation : $x_2^A = 2x_1^A$

$\Rightarrow \frac{p_1}{p_2} = 2$ (because tangency conditions requires $MRS^A = \frac{p_1}{p_2} = MRS^B$)

One line that goes through the endowment w and has slope -2

$$(x_2^A - 500) = -2(x_1^A - 50)$$

plug in point you know is on the line (50,500)

$\Rightarrow$ substitute $x_2^A = 2x_1^A$

$\Rightarrow$ $x^A = (150, 300)$

$x^B = (250, 500)$

Practice question 3: Non-interior solutions in exchange economies

(a) *Find all interior points on the contract curve*

Endowments w

1. $w^A = (11, 8)$
2. $w^B = (4, 8)$

Quasilinear preferences:

$$u^A(x^A) = 2\sqrt{x_1^A} + x_2^A$$

$$u^B(x^B) = 4\sqrt{x_1^B} + x_2^B$$

$$MRS^A = \frac{\frac{1}{\sqrt{x_1^A}}}{1}$$

$$MRS^B = \frac{\frac{2}{\sqrt{x_1^B}}}{1} = \frac{2}{\sqrt{15 - x_1^A}}$$

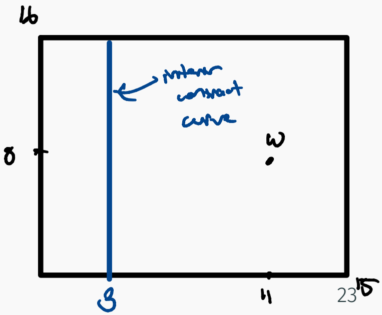
$\Rightarrow$ contract curve (interior points)

$$\Rightarrow \frac{1}{\sqrt{x_1^A}} = \frac{2}{\sqrt{15 - x_1^A}}$$

$$\Rightarrow \sqrt{15 - x_1^A} = 2\sqrt{x_1^A}$$

$$\Rightarrow 15 - x_1^A = 4x_1^A$$

$$\Rightarrow x_1^A = 3$$



Boundary considerations in exchange economies

(b) Argue the contract curve includes edges of the Edgeworth Box and connects the upper-right and lower-left corners

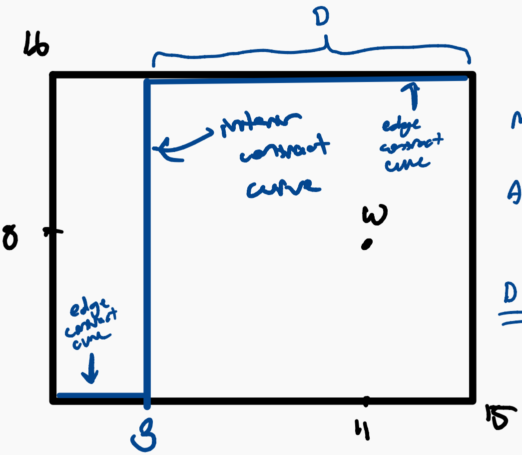
Endowments w

1. $w^A = (11, 8)$
2. $w^B = (4, 8)$

Quasilinear preferences:

$$u^A(x^A) = 2\sqrt{x_1^A} + x_2^A$$

$$u^B(x^B) = 4\sqrt{x_1^B} + x_2^B$$



C : $x_1^A < 3$, $x_2 = 0$

$MRS^A = \frac{1}{\sqrt{x_1^A}}$

$MRS^B = \frac{2}{\sqrt{15 - x_1^A}}$

As $x_1^A : 3 \to 0$

$MRS^A \uparrow \neq MRS^B \downarrow$

D : $x_1^A > 3$, $x_2 = 16$

as $x_1^A : 3 \to 15$

$MRS^A \downarrow \neq MRS^B \uparrow$

$\to$ no mutually beneficial exchange possible

Marshallian demand has corner solutions i.e. Pareto optimal ∴ part of contract curve

Boundary considerations in exchange economies

(c) Solve for a competitive equilibrium

Endowments w

1. $w^A = (11, 8)$
2. $w^B = (4, 8)$

Quasilinear preferences:

$$u^A(x^A) = 2\sqrt{x_1^A} + x_2^A$$

$$u^B(x^B) = 4\sqrt{x_1^B} + x_2^B$$

Assume an interior solution.

Then $MRS^A = \frac{p_1}{p_2} = MRS^B$

$$\Rightarrow \underbrace{\frac{1}{\sqrt{x_1^A}} = \frac{p_1}{p_2}}\ \underbrace{= \frac{2}{\sqrt{x_1^B}}}$$

$$\Rightarrow x_1^A(p, w^A) = p_2^2/p_1^2$$

$$x_1^B(p, w^B) = \frac{4p_2^2}{p_1^2}$$

$$x_1^A(p,w) + x_1^B(p,w) = w^A$$

$$\Rightarrow \frac{p_2^2}{p_1^2} + \frac{4p_2^2}{p_1^2} = 15$$

$$\Rightarrow \frac{5p_2^2}{p_1^2} = 15$$

$$\Rightarrow p_1/p_2 = 1/\sqrt{3}$$

At these prices,

$$x_1^A = \left(\frac{\sqrt{3}}{1}\right)^2 = 3$$

$$x_1^B = 4\left(\frac{\sqrt{3}}{1}\right)^2 = 12$$

$$\Rightarrow x_2^A = \frac{w_1^A p_1 + w_2^A p_2 - x_1^A p_1}{p_2}$$

numerator is wealth from initial endowment minus value of holdings of good 1 after exchange

denominator is p_2 since A's remaining wealth must be in holdings of good 2

$$= \frac{11 \cdot 1 + 8\sqrt{3} - 3}{\sqrt{3}} = \frac{8 + 8\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow x_2^B = \frac{w_1^B p_1 + w_2^B p_2 - x_1^B p_1}{p_2}$$

$$= \frac{8\sqrt{3} - 8}{\sqrt{3}}$$

$\therefore x_1^A, x_1^B, x_2^A, x_2^B$ all > 0

$\therefore$ a valid interior CE

3. Consider an economy with total endowments $w = (60, 40)$, and suppose both agents have symmetric perfect complement preferences. Argue that the contract curve is actually a 2-dimensional parallelogram area bound between the lines $x_1^A = x_2^A$ and $x_1^B = x_2^B$.

4. Suppose Alice has symmetric perfect complement utility and an endowment of $w^A = (200, 50)$ while Bob has symmetric perfect substitute utility and an endowment of $w^B = (0, 50)$. Show that the contract curve includes the line segment $x_1^A = x_2^A$ within the Edgeworth Box, but the contract curve does include not the upper right corner or any other points on the boundary of the Edgeworth Box. Show that $p_1 = 1$ and $p_2 = 2$ is a competitive equilibrium price.

   Any interior point can only be on the contract curve if the set of Pareto improvements is empty. With the given preferences, a quick sketch of indifference curves can reveal that such points are exactly along the line $x_1^A = x_2^A$. Any other interior point has a triangular region of Pareto improvements. In this case, the upper right corner is the allocation with $x^A = (200, 100)$ and $x^B = (0, 0)$, leaving agent A with a utility of 100 and agent B a utility of 0. Compare this outcome to the feasible allocation $x^A = (100, 100)$ and $x^B = (100, 0)$, which gives A a utility of 100 and B a utility of 100. Thus we have found an allocation that Pareto dominates the upper right corner, and therefore the upper right corner is not Pareto efficient in this case. This is due to the fact that A does not have *strongly* monotonic preferences.

   Now let's consider if prices are $p_1 = 2$ and $p_1 = 1$. With symmetric perfect substitute utility, B will always choose to spend their entire income on the cheapest good, which is good 1 in this case. Thus their demands will be $x_1^B = 2(50) = 100$ and $x_2^B = 0$. With symmetric perfect complement utility, A will always choose to spend their entire income such that $x_1^A = x_2^A$.

$$2x_1^A + x_2^A = 3x_1^A = 200 + 2(50) \Rightarrow x_1^A = x_2^A = 100$$