ECON-UN 3211 - Intermediate Microeconomics

Recitation 6: The Producer's Problem II - The Supply Decision

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Plan for today

Midterm feedback

Review of relevant concepts

The Producer's Problem II:
Profit maximization + supply choice

Practice questions

Midterm feedback

See recording of Recitation 7 for discussion of the questions emphasized here

Q1. Find a utility function representing the following preferences *(a) Jesabelle spends all of her income on the good with the lowest price*

- $u(x) = x_1 + x_2$
- Also valid: symmetric concave preferences like $u(x) = \max\{x_1, x_2\}$
- Some people wrote $u(x) = \alpha x_1 + \beta x_2$ but this only works when $\alpha = \beta$

Q1. Find a utility function representing the following preferences
(b) Carly always buys an equal number of units of good 1 and 2

- $u(x) = \min\{x_1, x_2\}$
- Some people wrote $u(x) = \min\{\alpha x_1, \beta x_2\}$ but this only works when $\alpha = \beta$

Q2. With two budget lines, illustrate the income and substitution effects when the price of good 1 increases and preferences are Cobb-Douglas

- Draw the correct budget lines
- Show the correct direction of change: a price increase in good 1 means any substitution effect will involve consuming less of good 1 and more of good 2
- Label the effects properly: it's the change in x_1 and x_2
- Capturing the zero cross-price effect of the Cobb-Douglas

Q4. Derive the Marshallian demand for preferences $u(x) = 12\sqrt{x_1} + x_2$

- Recognize this is an example of quasi-linear utility
- Our approch for these: assume an interior solution and see where it is feasible
- If nowhere, then we only have corner solutions
- If only in some places, then we will have piecewise demand and at least two cases (which is the case here)

Review of relevant concepts

The Producer's Problem I: Cost Minimization (Recitation 5)

$$\max(\text{Profit}) = \max(\text{Revenue} - \text{Cost}) = \max_q \{p(q)q - c(q)\}$$

Step 1: Cost minimization

$$\min_{\{x_1, x_2\}} w_1 x_1 + w_2 x_2$$
$$\text{s.t.} \quad f(x_1, x_2) \geq q$$

Given:

- technological constraint $f(x_1, x_2)$
- input prices w_1, w_2
- output quantity q

Derive:

- conditional factor demand functions

$$x_1^*(w_1, w_2, q)$$
$$x_2^*(w_1, w_2, q)$$

- cost function

$$c(q) = w_1 x_1^*(w_1, w_2, q) + w_2 x_2^*(w_1, w_2, q)$$

The Producer's Problem II: The Supply Decision

$$\max(\text{Profit}) = \max(\text{Revenue} - \text{Cost}) = \max_q \{p(q)q - c(q)\}$$

Step 2: The supply decision

$$\max_q \; p(q)q - c(q)$$
$$\text{s.t.} \quad q \geq 0$$

Given

- Consumer demand function $q(p)$
- Producers' cost function $c(q)$

Derive:

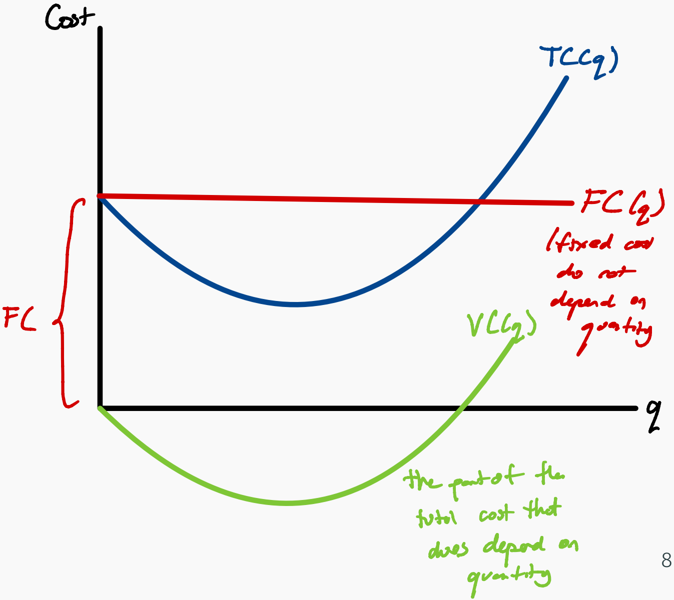
- Marginal revenue $MR(q)$
- Marginal cost $MC(q)$
- Profit-maximizing supply decision $q^* \geq 0$
- Maximal profit $\pi^* = p(q^*)q^* - c(q^*)$

Total costs

The cost function we've derived is called the total cost function, which can be decomposed as such:

$$TC(q) = FC + VC(q)$$

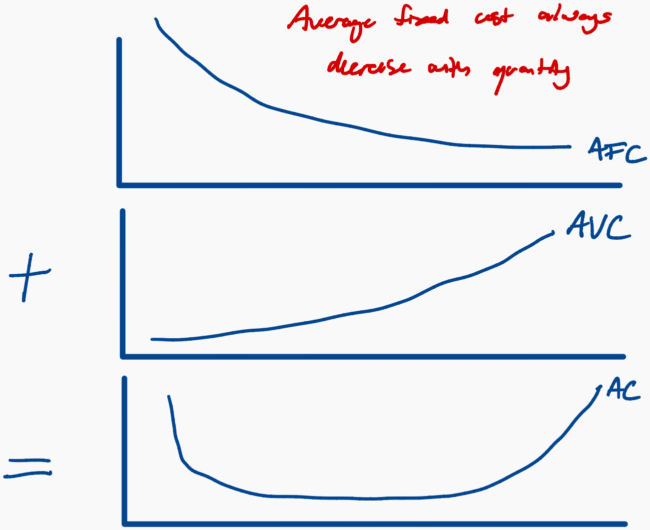
- Fixed costs FC
  - The costs that don't depend on quantity
  - "Fixed" in the short run
- Variable costs $VC(q)$
  - The costs that do depend on quantity
  - Can be varied in the short run



Average costs

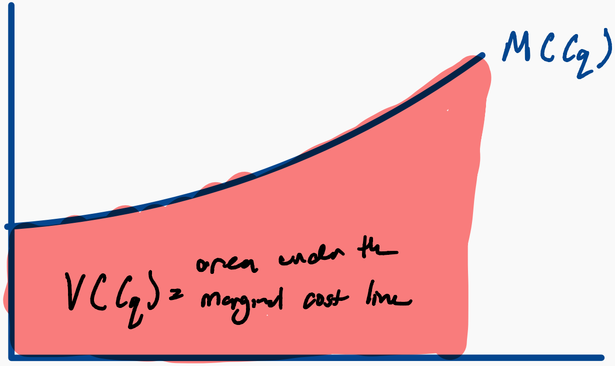
$$AC(q) = \frac{c(q)}{q}$$

- Cost function divided by quantity: the cost per unit of output
- Average cost = average fixed costs + average variable costs



Marginal costs

$$
\begin{aligned}
MC(q) &= \frac{dTC(q)}{dq} \\
&= \frac{dFC(q)}{dq} + \frac{dVC(q)}{dq} \\
&= 0 + \frac{dVC(q)}{dq} \\
&= \frac{dVC(q)}{dq}
\end{aligned}
$$



- Since fixed costs don't depend on quantity, fixed costs do not affect marginal costs
- Marginal costs are the first derivative of the variable cost

$$VC(q) = \int MC(q)\, dq$$

Relation between average costs and marginal costs

- For q such that $MC(q) < AC(q)$, then average costs are decreasing
- For q such that $MC(q) > AC(q)$, then average costs are increasing
- For q such that $MC(q) - AC(q)$, then average costs are at an inflection point (minimum/maximum)
- Some more in Chapter 22 but cannot fit into this recitation

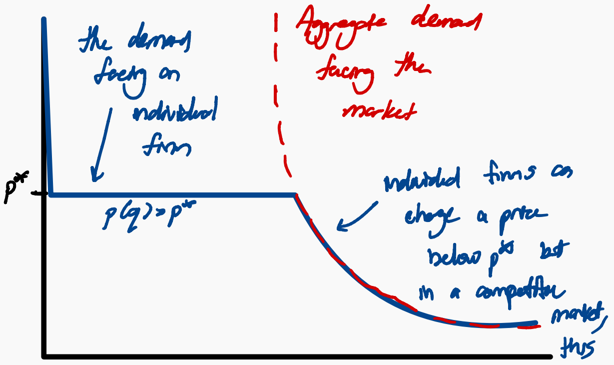
The Producer’s Problem II:
Profit maximization + supply choice

Market structure

$$\max_{q} p(q)q - c(q)$$

$$\text{s.t.} \quad q \geq 0$$

- $p(q)$ comes from inverting *the demand function $q(p)$ the firm faces*
- Whether or not this resembles *the market demand function* depends on market structure
- In a pure competitive market, the firm is a "price taker": $p(q) = p^*$
- The market supply function $q^S(p)$ is just the sum of all the individual firms' supply functions at a given p



Individual firms are "price takers": they cannot affect the price the market charges: they take $p(q) = p^*$ as given at all quantities

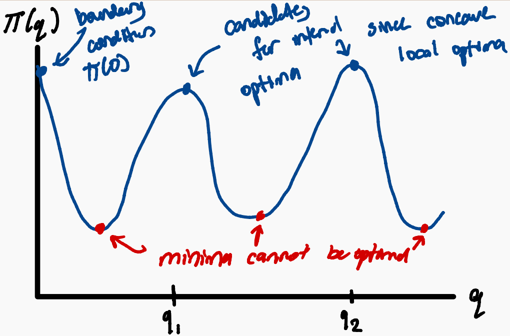
The supply decision

π(q), the profit function

$$\max_q \; p(q)q - c(q)$$

$$\text{s.t.} \quad q \geq 0$$

- Constrained optimization problem
- First-order condition for an internal solution equates marginal revenue with marginal cost
- Second-order condition requires local concavity of the profit function at the local optimum



So to determine supply choice q^*, need to compare

$\pi(0)$ vs. $\pi(q_1)$ vs. $\pi(q_2)$

Marginal revenue

Total revenue:

$$TR(q) = p(q)q$$

Marginal revenue: $MR(q) = \frac{dTR(q)}{dq}$

$= p'(q)q + p(q)\frac{dq}{dq}$ (by Chain Rule)

$= p'(q)q + p(q)$

price effect <0 quantity effect >0

- $p(q)$ is the quantity effect: the increase in revenue from selling an additional unit of output, which is also the price at quantity q
- $p'(q)q$ is the price effect: the decrease in revenue associated with having to lower the price on all previous units in order to be able to sell the qth unit

In recitation 7, we look at effect of changes in market structure: firms leave the market. This decreases supply which has both a price effect and quantity effect on producer surplus. We ask which effect is bigger

Marginal revenue

Total revenue:

$$TR(q) = p(q)q$$

Marginal revenue:

$$\begin{aligned} MR(q) &= \frac{dTR(q)}{dq} \\ &= \frac{dp(q)q}{dq} \\ &= \frac{dp(q)}{dq}q + p(q)\frac{dq}{dq} \\ &= p'(q)q + p(q) \end{aligned}$$

- In a perfectly competitive market, firms are price takers: $p(q) = p^*$ for all q
  - $p(q) = p^*$ for all q so the quantity effect is always $p^* > 0$
  - Since price is independent of quantity: $p'(q) = 0$ and the price effect is zero
- In an imperfectly competitive market, optimally supplying an additional good requires decreasing the market price to meet demand
  - $p'(q) < 0$ so the quantity effect is decreasing and the price effect is negative

The first-order condition

First-order condition:

$$\begin{aligned} MR(q) &= MC(q) \\ p'(q)q + p(q) &= c'(q) \end{aligned}$$

Under perfect competition,
$p(q) = p^*$ for all q (price-taking firms)
$\therefore p'(q) = \frac{dp^*}{dq} = 0$
$\therefore p(q) = p^* = c'(q)$: the marginal cost is just the price p^* under perfect competition

- The revenue gained by one more unit of output equals the cost of producing that additional unit
    - If MR > MC, then profitable to produce more
    - If MR < MC, then the last unit was produced at a loss
- Under pure competition, $p'(q) = 0$ and $p(q) = p^*$ so $MR(q) = p^* = c'(q)$
- Otherwise, $p(q) \geq p^*$ but $MR(q) = MC(q)$ still holds

The second-order condition

Second-order condition:

$$\frac{d^2\{p(q)q - c(q)\}}{dq^2} < 0$$

concavity of the profit function

$$\Rightarrow \frac{d}{dq}\left[p'(q)q + p(q) - c'(q)\right] < 0,$$ the derivative of the first derivative of the profit function

$$\Rightarrow p''(q)\cdot q + p'(q)\cdot 1 + p'(q) - c''(q) < 0$$

$$\Rightarrow p''(q)\cdot q + 2p'(q) - c''(q) < 0$$

(continued on next slide)

- At an internal optimal point, the profit function must be concave
- For a competitive firm, this amounts to marginal cost being an increasing function

The second-order condition (for a competitive firm): $\frac{d^2\{p(q)q - c(q)\}}{dq^2} < 0 \Leftrightarrow c''(q) > 0$

For a competitive firm, $p(q) = p^*$ for all q

$$\Rightarrow p'(q) = 0$$

$$\Rightarrow p''(q) = 0$$

So plugging into the expression on previous slide:

$$\underbrace{p''(q)}_{=0} \cdot q + 2\underbrace{p'(q)}_{=0} - c''(q) < 0$$

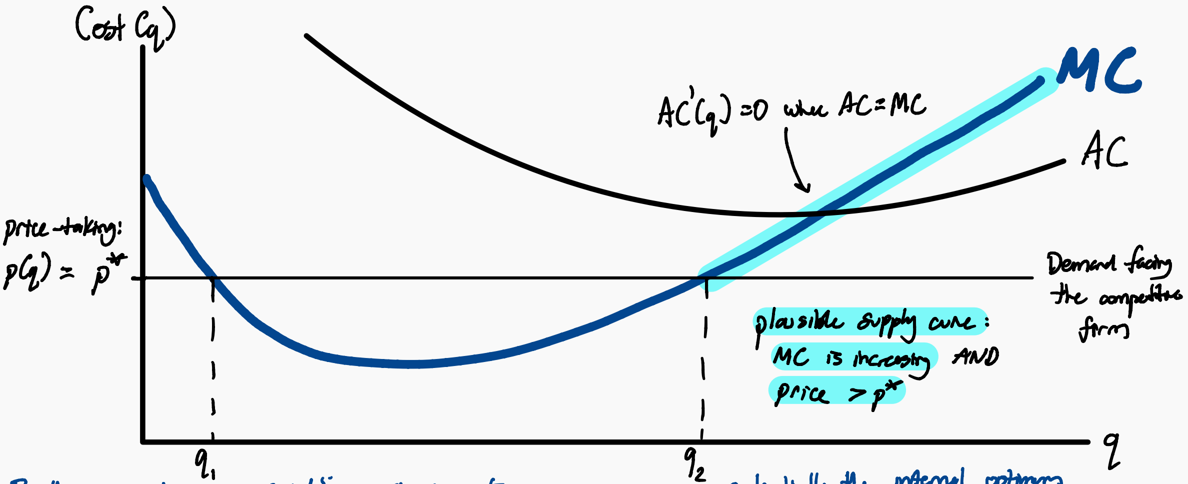
$$\Rightarrow -c''(q) < 0$$

$$\Rightarrow c''(q) > 0$$

or equivalently, $\frac{d}{dq} MC(q) > 0$

i.e. The second-order condition requires the profit function to be concave at any internal profit-maximizing supply choice. Under perfect competition, this occurs whenever the cost function is convex (which also means marginal cost is increasing)

The second-order condition (for a competitive firm): $\frac{d^2\{p(q)q - c(q)\}}{dq^2} < 0 \Leftrightarrow c''(q) > 0$



By the second-order conditions, q_1 cannot be the optimal supply choice since MC is decreasing at q_1

~ potentially the internal optimum since MC is increasing at q_2

The boundary condition: profit when $q = 0$

- Computing local maxima give us candidates for the global maximum
- But we also need to check profit levels when $q = 0$
- If a firm produces zero output, it still has to pay fixed costs F so profits at $q = 0$ are -F
- Profits from producing at local maximum $\hat{q} > 0$ are

$$\pi(\hat{q}) = p(\hat{q})\hat{q} - VC(\hat{q}) - F$$

- So producing zero output is more profitable when

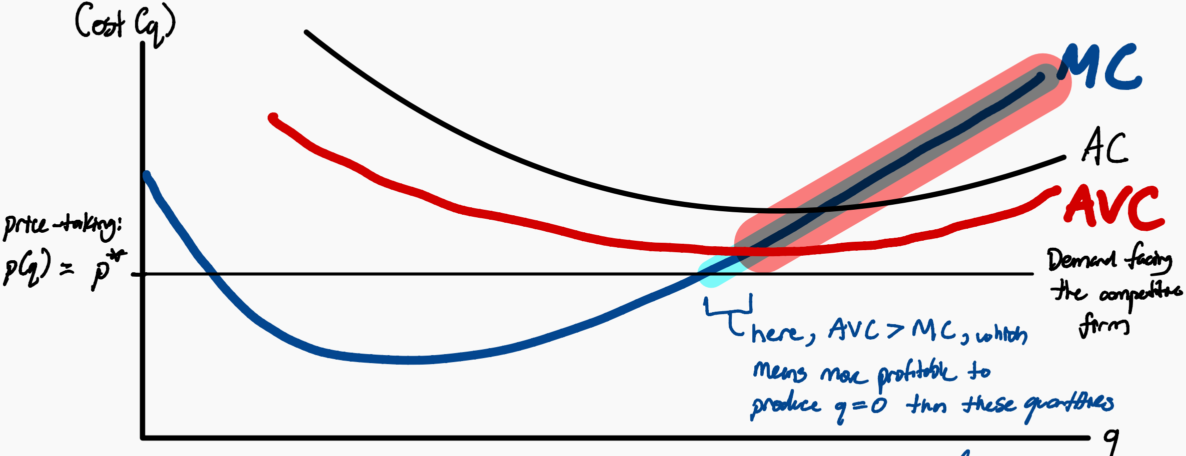
$$\begin{aligned} -F &> p(\hat{q})\hat{q} - VC(\hat{q} - F \\ \Rightarrow \frac{VC(\hat{q})}{\hat{q}} &> p \\ \Rightarrow \text{AVC}(\hat{q}) &> p = \frac{VR(q)}{q} \end{aligned}$$

(average variable revenue)

This is the "shutdown condition"

- Only when the marginal cost curve is above the AVC curve is it profitable to produce positive q

The boundary condition: profit when $q = 0$



Supply curve is red: firm will only supply at quantities where

i) MC is increasing (second-order condition)

ii) $MC \geq p^*$ (positive profit)

iii) MC > AVC (boundary condition)

Practice questions

Solutions given in annotated Recitation 7 slides

1. A firm faces cost function $c(q) = 10 + 2q^2$ and demand $q = 600 - \frac{p}{2}$

(a) What is the optimum supply choice?

1. A firm faces cost function $c(q) = 10 + 2q^2$ and demand $q = 600 - \frac{p}{2}$
(b) What is the resulting price?

1. A firm faces cost function $c(q) = 10 + 2q^2$ and demand $q = 600 - \frac{p}{2}$
(c) What profit will the firm make?

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(a) What is the aggregate demand in this market?

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(b) What is the aggregate supply in this market?

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(c) Find the competitive equilibrium

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(d) What is consumer/producer surplus?

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are ~~50~~ **10** identical firms

(e) What is the new equilibrium?

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are ~~50~~ **10** identical firms

(f) What is the new consumer/producer surplus?

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are ~~50~~ **10** identical firms

(g) Do these changes in surplus make sense?

- The consumer population has not changed but prices have increased
- Thus, fewer consumers are being served so we should expect a decrease in CS
- There are fewer firms but prices have increased so effect on producer surprlus ambiguous
- However, prices have increased more than the quantity has decreased so total producer surplus has increased
- Each individual firm experiences an even more significant increase in surplus

3. Taxes, subsidies, and deadweight loss
(a) Calculate the deadweight loss of a $300 tax per unit on consumers

- $Q^D(p) = 12000 - 10p$
- $Q^S(p) = 15p$

Example 3: Taxes, subsidies, and deadweight loss
(b) Calculate the deadweight loss of a \$300 subsidy per unit to producers

- $Q^D(p) = 122000 - 10p$
- $Q^S(p) = 15p$