ECON-UN 3211 - Intermediate Microeconomics

Recitation 4: Hicksian price effect decomposition and consumer welfare

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Some housekeeping

- New on Courseworks: 00-RunningUpdates.pdf
- Pset 4 grades coming back this weekend
- Varian textbook (with disclaimers)

Review of relevant concepts

Comparative statics of income m: budget increases outwardly in parallel

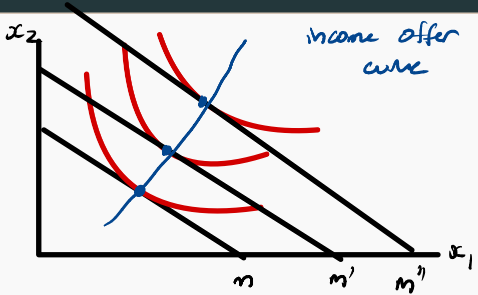
- **Normal goods**: goods you demand more of as income increases

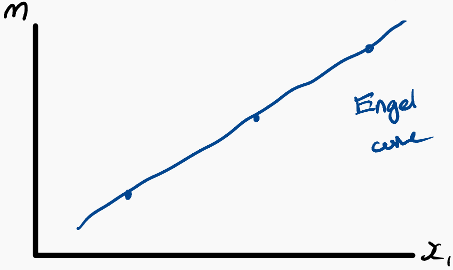
  $x(p, m)$

  - Luxury goods — demand increases more than proportionally
  - Necessary goods

  {
  elasticities

- Inferior goods: goods you demand less of as income increases
- Graphical relationships:
  1. The ***income* offer curve**: traces how optimal bundle changes as income changes in $x_1 - x_2$ space
  2. The **Engel curve**: traces how optimal demand for a good changes in $x_1 - m$ space

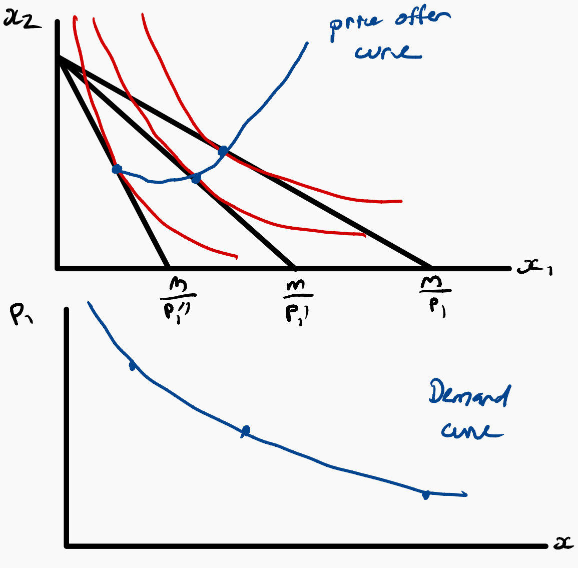




Comparative statics of own price p_1: budget increases outwardly along one axis

Change in demand of **good 1** in response to a change in its own price p_1

- ***Price* offer curve**: traces the bundles that would be demanded at different prices of good 1 in $x_1 - x_2$ space
- **Demand curve**: traces how optimal demand for a good changes in $x_1 - p_1$ space



Slutsky decomposition of the demand effects of a price change ($p^0 \to p'$)

There are two reasons a price change would affect consumer's behavior:

1. Substitution effect: goods becoming more expensive incentivizes me to consume more of the other good (change in price ratio generally means original bundle doesn't satisfy tangency condition)
2. Income effect: a good becoming more (less) expensive is in a sense equivalent to becoming poorer (richer)

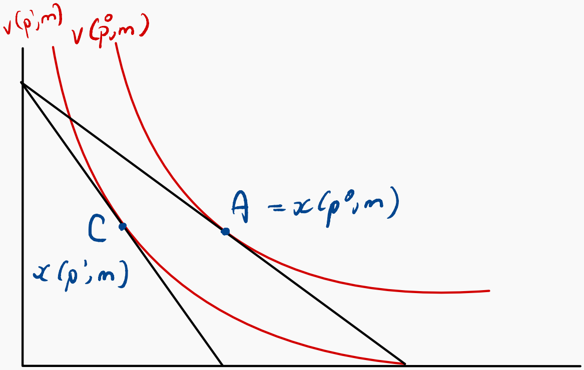
We decompose the total price effect on demand $x(p^0, m) - x(p', m)$

1. **Holding fixed purchasing power**, how much of the change in demand is due to changes in how the market values the two goods?
2. Holding fixed how the market values the two goods, how much of the change in demand is due to **changes in purchasing power**?

The Slutsky vs. Hicksian approaches to decomposing the total price effect

Depicting the price effect

- At original prices p^0 and income (p^0, m), the consumer demands bundle $A = x(p^0, m)$. This gives indirect utility $v(p^0, m)$
- At new prices p' and no change in nominal income m, the consumer demands bundle $C = x(p', m)$. This gives indirect utility $v(p^1, m)$, which is (weakly) higher for price decreases and (weakly) lower for price increases.



The Slutsky vs. Hicksian approaches to decomposing the total price effect

We decompose the total price effect on demand $x(p', m) - x(p^0, m)$ (C-A)

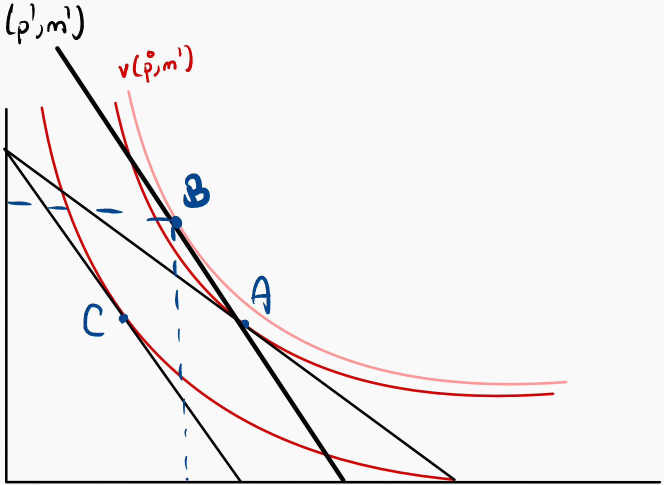
1. **Holding fixed purchasing power**, how much of this change in demand is due to changes in how the market values the two goods?
2. Holding fixed how the market values the two goods, how much of this change in demand is due to **changes in purchasing power**?

But what is "purchasing power"?

- Slutsky: compensate the consumer ($m \to m'$) after the price change just enough that they can afford the same bundle $x(p^0, m)$
- Hicks: compensate the consumer ($m \to m''$) after the price change just enough that they can access the same level of utility $v(p^0, m)$

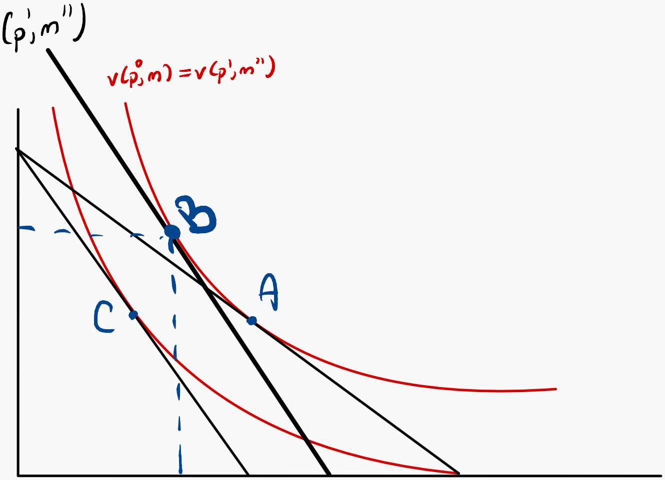
The Slutsky vs. Hicksian approaches to decomposing the total price effect

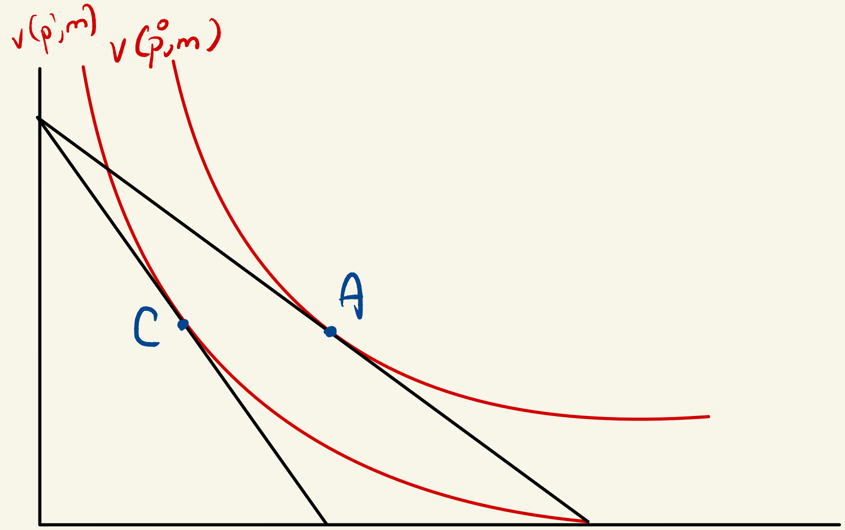
- **Slutsky**: Parallel shift in new budget line (p', $m \to m'$) until bundle A is affordable
- Equivalently, pivot original budget line about bundle A until it is parallel to the new price ratio
- Bundle B: $x(p', m')$
- Substitution effect (B-A): $x(p', m') - x(p^0, m)$
- Income effect (C-B): $x(p', m) - x(p', m'')$



The Slutsky vs. Hicksian approaches to decomposing the total price effect

- **Hicks**: Parallel shift in new budget line ($p', m \to m''$) until $v(p^0, m)$ is accessible
- Equivalently, rotate original budget line about indifference curve $v(p^0, m)$ until it is parallel to the new price ratio
- Bundle B: $h(p', \bar{u} = v(p^0, m) \equiv v(p', m''))$
- Substitution effect (B-A): $h(p', \bar{u}) - x(p^0, m)$
- Income effect (C-B): $x(p', m) - h(p', \bar{u})$

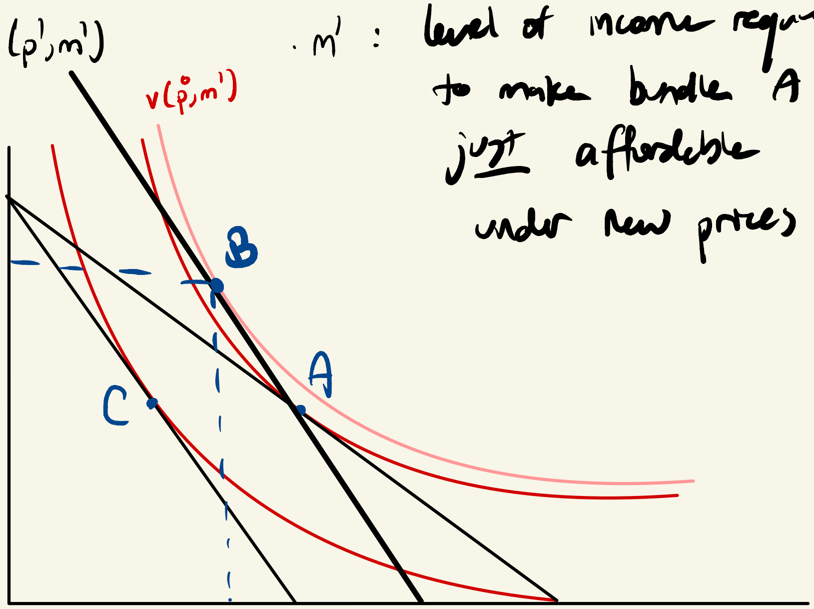




$A \to B$: the substitution effect

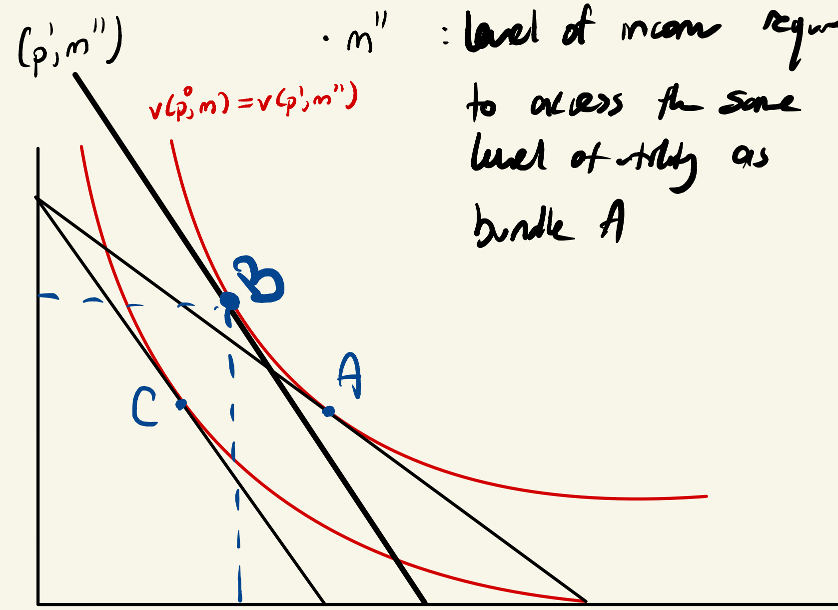
$B \to A$: the income effect of $m \to m'$

Compensation under Slutsky



· m' : level of income required to make bundle A *just* affordable under new prices

Compensation under Hicks



· m'' : level of income required to access the same level of utility as bundle A

Slutsky m' vs. Hicksian m'' compensation

Slutsky compensation:

$$
\begin{aligned}
m &= p_1^0 x_1(p^0, m) + p_2^0 x_2(p^0, m) \\
m' &= p_1' x_1(p^0, m) + p_2' x_2(p^0, m) \\
\Rightarrow m' - m &= \Delta p_1 x_1(p^0, m) + \Delta p_2 x_2(p^0, m)
\end{aligned}
$$

Hicks compensation:

$$
\begin{aligned}
m'' &= e(p', \overline{u}) \\
&= e(p', v(p^0, m)) \\
\Rightarrow m'' - m &= e(p', \overline{u}) - m \\
&= e(p', \overline{u}) - e(p^0, \overline{u})
\end{aligned}
$$

This quantity $m'' - m$ is also called the **compensating variation** (more later)

The Hicksian approach bridges Marshallian (UMP) and Hicksian (EMP) concepts

Δx^n + Δx^s

income effect   substitution effect

$$
\begin{aligned}
x(p', m) - x(p^0, m) &= [x(p', m) - h(p', \overline{u})] + [h(p', \overline{u}) - x(p^0, m)] \\
&\equiv [x(p', m) - h(p', v(p^0, m))] + [h(p', v(p^0, m)) - x(p^0, m)] \\
&\equiv [x(p', m) - h(p', v(p', m''))] + [h(p', v(p', m'')) - x(p^0, m)]
\end{aligned}
$$

General process for calculating the two effects (Hicksian approach)

1. Using preferences, derive the general Marshallian demand function $x^*(p, m)$ and indirect utility function $v(p, m)$
2. Using preferences, derive the general Hicksian demand function $h(p, m)$
3. Calculate Marshallian for initial conditions (p^0, m), call this bundle A: $x^*(p^0, m)$
4. Calculate Marshallian for new conditions (p', m), call this bundle C: $x^*(p^0, m)$
5. Calculate utility of bundle A: $\overline{u} := v(p^0, m)$
6. Plug new price and this utility into the Hicksian demand function to get bundle B: $h(p', \overline{u})$
7. As before, difference between bundles B and A is the substitution effect Δx^s
8. The remainder (difference between bundles C and B) is the income effect Δx^n

Welfare

Welfare effects of an environmental change

1. We want to be able to quantify the welfare effects of a change in income or prices)
2. A natural way is to look at the effect on utility. For example, for a price change $p^0 \to p'$
$$\Delta\bar{u} := v(p', m) - v(p, m)$$
3. But utility is not interpretable: what does a loss of five utility mean? And the same preferences can be represented by infinite utility functions; five utility under one is equivalent to a billion utility under another
4. We want to express welfare effects in interpretable units, namely in terms of money/income
5. An income change is easy: a change of income from m to m' is a change in welfare of $m' - m$
6. Price effects, as we've seen, are a bit more complicated

Welfare effects of a price change $p^0 \to p'$

- We want to be able to express the welfare effects of an environmental change (change in income or prices)
- One way is to look at the change in utility. For example, for a price change $p^0 \to p'$

$$\Delta \bar{u} := v(p', m) - v(p^0, m)$$

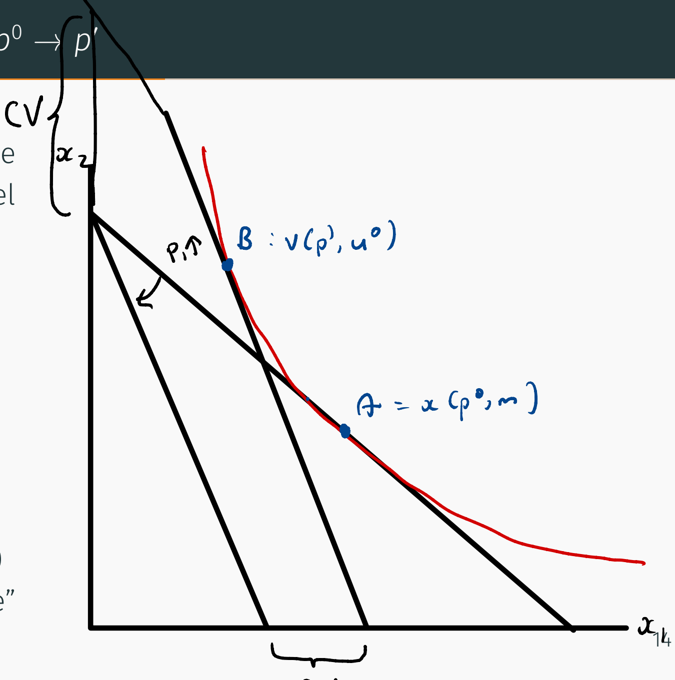
- But utility is not interpretable: what does a loss of five utility mean? And the same preferences can be represented by infinite utility functions; five utility under one is equivalent to a billion utility under another
- We want to express welfare effects in interpretable units, namely in terms of money/income
- An income change is easy: a change of income from m to m' is a change in welfare of $m' - m$
- We want to express price effects on welfare in terms of income too

Welfare effects of a price change $p^0 \rightarrow p'$

Compensating variation: “the additional income necessary under the new prices to achieve the original level of utility”

$$\begin{aligned} CV &:= m'' - m \\ &= e(p', \overline{u}) - m \\ &= e(p', \overline{u}) - e(p^0, \overline{u}) \\ &= \underbrace{e(p', v(p^0, m))}_{= v(p', m'')} - e(p^0, v(p^0, m)) \end{aligned}$$

Alternatively, “how much a consumer would need to be paid (compensated) to be made whole by the price change” (e.g., dividend for a carbon tax)

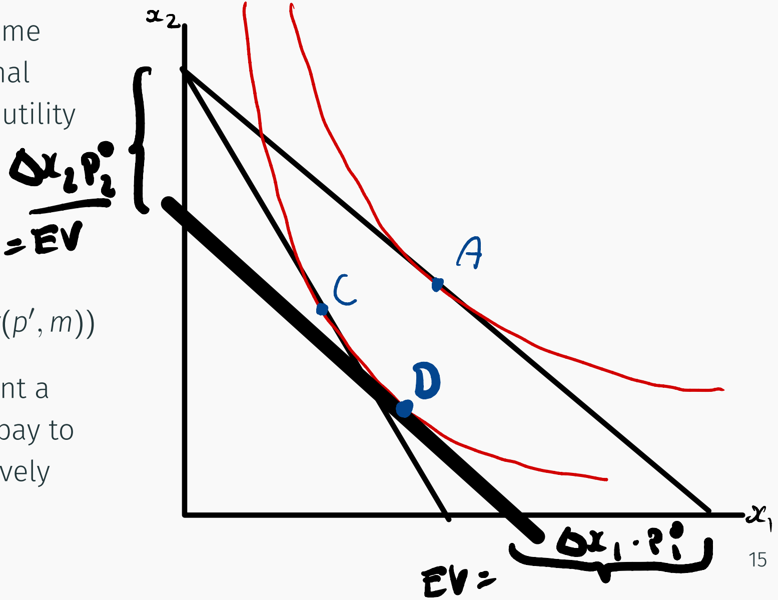


Welfare effects of a price change $p^0 \to p'$

Equivalent variation: "the income decrease required under original prices to experience the same utility loss as from the price change"

$$
\begin{aligned}
EV &:= m - e(p^0, \overline{u}^1) \\
&= e(p^0, \overline{u}^0) - e(p^0, \overline{u}^1) \\
&= e(p^0, v(p^0, m)) - e(p^0, v(p', m))
\end{aligned}
$$

Alternatively, "maximum amount a consumer would be willing to pay to avoid the price change" (positively negative for a price decrease)



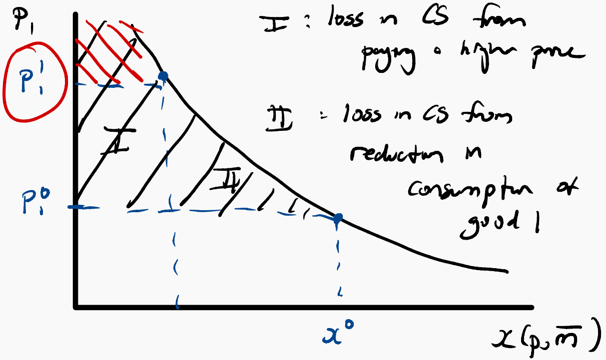
Welfare effects of a price change $p^0 \to p'$

Consumer surplus: "the monetary value of consuming at a price lower than the max one is willing to pay"

- Area below the demand curve and above the market price is the consumer's surplus

$$\Delta CS := \int_{p_i^0}^{p_i'} x_i(p, m)dp_i$$

1. Loss in surplus from paying a higher price per unit of the good
2. Loss in surplus from consuming a lower quantity



Example questions

Setup

- $u(x) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$
- $m = 100$
- $p^0 = (1, 1)$
- $p' = (5, 1)$

Questions

1. **Calculate Marshallian demands at both prices**
2. Calculate the substitution effect
3. Calculate the income effect

1. Recall for Cobb-Douglas functions $u(x) = x_1^{\alpha} x_2^{\beta}$, we have general Marshallian demands

$$x^*(p,m) = \left(\frac{\left(\frac{\alpha}{\alpha+\beta}\right) m}{p_1}, \frac{\frac{\beta}{\alpha+\beta} m}{p_2} \right)$$

So for this utility function where $\alpha = \beta = 1/2$, we have

$$x^*(p,m) = \left(\frac{m}{2p_1}, \frac{m}{2p_2} \right)$$

Therefore,

$$x^0 = x^*((1,1),100) = \left(\frac{100}{2}, \frac{100}{2} \right) = (50,50)$$

$$x' = x^*((5,1),100) = \left(\frac{100}{10}, \frac{100}{2} \right) = (10,50)$$

1. Hicksian approach to price effects

Setup

- $u(x) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$
- $m = 100$
- $p^0 = (1, 1)$
- $p' = (5, 1)$

Questions

1. Calculate Marshallian demands at both prices
2. **Calculate the substitution effect**
3. Calculate the income effect

2. $\Delta x^s = h(p', \bar{u}^0) - x(p^0, m)$

Need to derive Hicksian demand functions

i) First, indirect utility

$$v(p,m) = u(x^*(p,m))$$

$$= \left(\frac{m}{2p_1}\right)^{1/2} \left(\frac{m}{2p_2}\right)^{1/2} = \frac{m}{2\sqrt{p_1 p_2}}$$

ii) Rearrange $v(p,m)$, solving for $m \equiv e(p,\bar{u})$, $\bar{u} \equiv v(p,m)$

$$e(p,u) \equiv m = 2\bar{u}\sqrt{p_1 p_2}$$

iii) Then apply Shephard's Lemma to get $h(p,\bar{u})$:

$$h(p,\bar{u}) = \left(\frac{\partial e(p,u)}{\partial p_1}, \frac{\partial e(p,u)}{\partial p_2}\right)$$

$$= \left(2\bar{u}\sqrt{p_2} \cdot \tfrac{1}{2} \cdot p_1^{-1/2}, 2\bar{u}\sqrt{p_1} \cdot \tfrac{1}{2} p_2^{-1/2}\right)$$

$$= \left(\bar{u}\sqrt{p_2/p_1}, \bar{u}\sqrt{p_1/p_2}\right)$$

1. Hicksian approach to price effects

Setup

- $u(x) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$
- $m = 100$
- $p^0 = (1, 1)$
- $p' = (5, 1)$

Questions

1. Calculate Marshallian demands at both prices
2. **Calculate the substitution effect**
3. Calculate the income effect

Then use $v(p^0; m)$ to get initial level of utility:

$$v(p^0; m) = u(x^0)$$
$$= u(50, 50)$$
$$= \sqrt{50}\sqrt{50} = 50$$

Then plug this and new prices into the Hicksian demand:

$$h(p', \bar{u}^0) = \left(\bar{u}^0 \sqrt{p_2'/p_1'}, \bar{u}^0 \sqrt{p_1'/p_2'}\right)$$
$$= \left(50\sqrt{1/5}, 50\sqrt{5/1}\right)$$
$$= \left(\frac{50}{\sqrt{5}}, 50\sqrt{5}\right)$$

Then $\Delta x^s = h(p', \bar{u}^0) - x(p^0; m)$

$$= \left(\frac{50}{\sqrt{5}} - 50, 50\sqrt{5} - 50\right)$$
$$= \left(50\left(\frac{1}{\sqrt{5}} - 1\right), 50\left(\sqrt{5} - 1\right)\right)$$

1. Hicksian approach to price effects

Setup

- $u(x) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$
- $m = 100$
- $p^0 = (1, 1)$
- $p' = (5, 1)$

Questions

1. Calculate Marshallian demands at both prices
2. **Calculate the substitution effect**
3. Calculate the income effect

3. $\Delta x^n = x(p', m) - h(p', \bar{u}^0)$

$= \left(10 - \frac{50}{\sqrt{5}}, 50 - 50\sqrt{5}\right)$

$= \left(10\left(\frac{\sqrt{5}-5}{\sqrt{5}}\right), 50\left(1-\sqrt{5}\right)\right)$

2. Welfare under perfect complements preferences

Setup

- $u(x) = \min\{x_1, 2x_2\}$
- $m = 480$
- $p^0 = (2, 2)$
- $p' = (2, 4)$

Questions

1. **Solve for the compensating variation**
2. Solve for the equivalent variation
3. Solve for the change in consumer surplus in good 1

1. CV

i) First, get Marshallin demand and calculate $x(p^0, m)$, $x(p', m)$

Complementarity ratio : $x_1 = 2x_2$

Budget constraint : $p_1 x_1 + p_2 x_2 = m$

$\Rightarrow p_1(2x_2) + p_2 x_2 = m$

$\Rightarrow x_2^*(p, m) = \frac{m}{2p_1 + p_2} \Rightarrow$ Plug into CR $\Rightarrow x_1^* = 2x_2^*$

$\Rightarrow$ Marshallin demand

$$x^*(p, m) = \left(\frac{2m}{2p_1 + p_2}, \frac{m}{2p_1 + p_2}\right)$$

$\Rightarrow$ Bundle $A := x(p^0, m) = \left(\frac{960}{6}, \frac{480}{6}\right) = (160, 80)$

Bundle $C := x(p', m) = \left(\frac{960}{8}, \frac{480}{8}\right) = (120, 60)$

2. Welfare under perfect complements preferences

Setup

- $u(x) = \min\{x_1, 2x_2\}$
- $m = 480$
- $p^0 = (2, 2)$
- $p' = (2, 4)$

Questions

1. **Solve for the compensating variation**
2. Solve for the equivalent variation
3. Solve for the change in consumer surplus in good 1

ii) Then derive expenditure function

$$v(p,m) = u(x^*(p,m)) = \min\left\{\frac{2m}{2p_1+p_2}, \frac{2m}{2p_1+p_2}\right\}$$

$$= \frac{2m}{2p_1+p_2}$$

Then $\bar{u}^0 = \frac{2m}{2p_1^0+p_2^0} = \frac{960}{4+2} = 160$

Isolate $v(p,m)$ for $m \equiv e(p,u)$ and $\bar{u} = v(p^0,m)$

$$\Rightarrow \bar{u} = \frac{2e(p,u)}{2p_1+p_2}$$

$$\Rightarrow e(p,\bar{u}) = \frac{\bar{u}}{2}(2p_1+p_2)$$

Setup

- $u(x) = \min\{x_1, 2x_2\}$
- $m = 480$
- $p^0 = (2, 2)$
- $p' = (2, 4)$

Questions

1. **Solve for the compensating variation**
2. Solve for the equivalent variation
3. Solve for the change in consumer surplus in good 1

iii) $CV := e(p', \bar{u}^0) - m$

$= \frac{\bar{u}^0}{2}(2p_1' + p_2') - m$

$= \frac{160}{2}(2 \cdot 2 + 4) - 480$

$= 80 \cdot 8 - 480$

$= 160$

Setup

- $u(x) = \min\{x_1, 2x_2\}$
- $m = 480$
- $p^0 = (2, 2)$
- $p' = (2, 4)$

Questions

1. Solve for the compensating variation
2. **Solve for the equivalent variation**
3. Solve for the change in consumer surplus in good 1

$$EV := m - e(p^0, \bar{u}')$$

$$\bar{u}' = v(p', m) = \frac{2m}{2p_1' + p_2'} = \frac{960}{2 \cdot 2 + 4} = 120$$

$$\Rightarrow EV = m - e((2,2), 120) = 480 - \left(\frac{120}{2}(2 \cdot 2 + 2)\right) = 480 - 360 = 120$$

2. Welfare under perfect complements preferences

Setup

- $u(x) = \min\{x_1, 2x_2\}$
- $m = 480$
- $p^0 = (2, 2)$
- $p' = (2, 4)$

Questions

1. for the compensating variation
2. Solve for the equivalent variation
3. **Solve for the change in consumer surplus in good 1**

$$\Delta CS = \int_{p_2^0}^{p_2'} x_2^*(p, m)\, dp_2$$

$$= \int_2^4 \frac{m}{2p_1 + p_2}\, dp_2$$

$$= m \left[\log(2p_1 + p_2) - \log(2p_1 + p_2)\right]_2^4$$

Evaluated for $m = 480$, $p_1 = 2$

$$= 480 \left[\log(4+4) - \log(4+2)\right]$$

$$= 480 \left[\log(8) - \log(6)\right]$$

$$= 480 \log(4/3)$$

$$\approx 138.09$$

Setup

- $u(x) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$
- $m = 2400$
- $p^0 = (4, 1)$
- $p' = (2, 1)$

Questions

1. **Solve for the compensating variation**
2. Solve for the equivalent variation
3. Solve for the change in consumer surplus in good 1

1. From Cobb-Douglas formula, we know

$$x^*(p,m) = \left(\frac{m}{4p_1}, \frac{3m}{4p_2}\right)$$

$$\therefore A := x(p^0, 2400) = (150, 1800)$$

$$C := x(p', 2400) = (300, 1800)$$

$$\Rightarrow \bar{u}^0 = u(150, 1800) = 150^{1/4} \, 1800^{3/4} = 1200$$

$$\bar{u}' = u(300, 1800) = 1200 \cdot 2^{1/4} \approx 1427.05$$

Thus $v(p,m) = v(x^*(p,m)) = \left(\frac{m}{4p_1}\right)^{1/4} \left(\frac{3m}{4p_2}\right)^{3/4}$

$$= \frac{3m}{(4p_1)^{1/4} (4p_2)^{3/4}}$$

$$\Rightarrow e(p, \bar{u}) = \frac{\bar{u}\left[(4p_1)^{1/4} (4p_2)^{3/4}\right]}{3}$$

3. Welfare under Cobb-Douglas preferences

Setup

- $u(x) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$
- $m = 2400$
- $p^0 = (4, 1)$
- $p' = (2, 1)$

Questions

1. **Solve for the compensating variation**
2. Solve for the equivalent variation
3. Solve for the change in consumer surplus in good 1

$$e(p, \bar{u}) = \frac{\bar{u}\left[(4p_1)^{1/4}(4p_2)^{3/4}\right]}{3}$$

$$CV := e(p', \bar{u}^0) - m$$

$$= \frac{1200}{3}\left[(4 \cdot 2)^{1/4}(4 \cdot 1)^{3/4}\right] - 2400$$

$$= 400 \cdot 8^{1/4} \cdot 4^{3/4} - 2400$$

$$\approx -497.27$$

Setup

- $u(x) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$
- $m = 2400$
- $p^0 = (4, 1)$
- $p' = (2, 1)$

Questions

1. Solve for the compensating variation
2. **Solve for the equivalent variation**
3. Solve for the change in consumer surplus in good 1

$$e(p, \bar{u}) = \frac{\bar{u}\left[(4p_1)^{1/4}(4p_2)^{3/4}\right]}{3}$$

2. $EV := m - e(p^0, \bar{u}')$

$$= 2400 - \frac{1427.05}{3}\left[(4 \cdot 4)^{1/4}(4 \cdot 1)^{3/4}\right]$$

$$\approx -290.871$$

Setup

- $u(x) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$
- $m = 2400$
- $p^0 = (4, 1)$
- $p' = (2, 1)$

Questions

1. for the compensating variation
2. Solve for the equivalent variation
3. **Solve for the change in consumer surplus in good 1**

3. $x^*(p,m) = \left(\frac{m}{4p_1}, \frac{3m}{4p_2}\right)$

$$\Rightarrow \Delta CS = \int_{p_0}^{p_1} x_1^*(p,m)\, dp_1$$

$$= \int_4^2 \frac{m}{4 p_1}\, dp_1$$

$$= \frac{m}{4}\left[\log p_1\right]_4^2$$

Evaluated at $m = 2400$ (p_2 does not appear for Cobb-Douglas)

$$= 600\left[\log 2 - \log 4\right] = 600 \log 0.5$$

$$\approx -415.89$$