ECON-UN 3211 - Intermediate Microeconomics

Recitation 3: Comparative statics and the Slutsky decomposition

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Some housekeeping

- Correction on previous week's notes: what I called Engel curves are actually the income offer curve; we'll explore this today
- Folder for blank versions of these slides
- Recordings policy: will post recordings to my recitation folder on Thursdays after the homework deadline

Review of relevant concepts

The utility maximization problem and the expenditure-minimization problem

$$\max_{\{x\}} u(x)$$
$$\text{s.t.} \quad p \cdot x \leq m$$

Solutions

- Marshallian demand $x^*(p, m)$
- Indirect utility $v(p, m) := u(x^*(p, m))$
- Roy's identity: $v(p, m) \Rightarrow x^*$

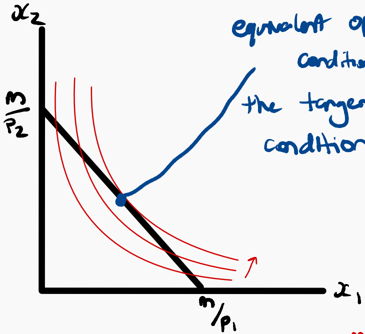
$$\min_{\{x\}} p \cdot x$$
$$\text{s.t.} \quad u(x) \geq \overline{u}$$

Solutions

- Hicksian demand $x^h(p, \overline{u})$
- Expenditure $e(p, \overline{u}) := p \cdot x^h(p, \overline{u})$
- Shephard's lemma: $e(p, \overline{u}) \Rightarrow x^h$

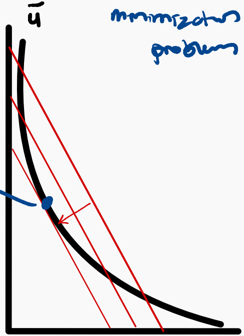
The utility maximization problem and the expenditure-minimization problem

The utility maximization problem



equivalent optimality condition: the tangency condition

The expenditure minimization problem



Given budget m, what's the highest indifference curve / utility I can afford to access?

Given desired level of utility $\bar{u}$ which has corresponding indifference curve in black, what's the smallest budget that still lets me access it?

The utility maximization problem and the expenditure-minimization problem

- Tangency condition: $MRS = \frac{p_1}{p_2}$ comes directly from preferences and prices which are the same in both problems
- Then facing given prices p, optimization is a function of budget m for utility maximization or choice of $\overline{u}$ for expenditure minimization
- Woodchuck's identities describe when these overlap:
    1. $v(p, e(p, \overline{u})) = \overline{u}$, plug in minimal expenditure for budget constraint
    2. $e(p, v(p, m)) = m$, plug in maximized utility for target level of utility
- Going from Marshallian (demand or utility) to Hicksian (demand or expenditure) or vice versa: use Woodchuck's and Roy's identity and/or Shephard's lemma

The utility maximization problem and the expenditure-minimization problem

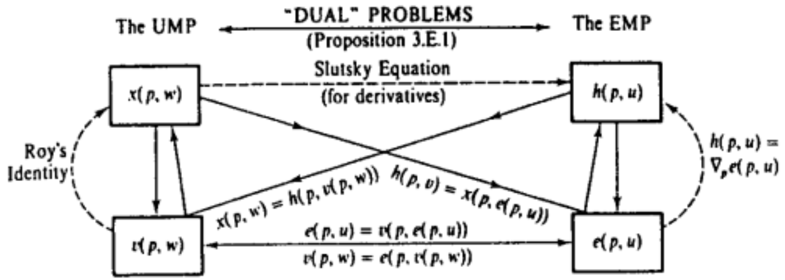


Figure 3.G.3

Relationships between the UMP and the EMP.

- w to refer to budget m
- h to refer to Hicksian demand x^h
- The unlabeled expression on the right is Shephard's Lemma
- Today: the Slutsky decomposition of income and substitution effects

Comparative statics of demand

Comparative statics of demand

Comparative statics: how does the optimal outcome change as we increase/decrease one parameter and keep all others the same (i.e., static)?

- $\frac{\partial x_1^*(p_1,p_2,m)}{\partial m}$: comparative statics of income
  - Normal goods
  - Inferor goods
- $\frac{\partial x_1^*(p_1,p_2,m)}{\partial p_1}$: comparative statics of own price p_1
  - Ordinary goods (obey the “law” of demand)
  - Giffen goods (rare)
- $\frac{\partial x_1^*(p_1,p_2,m)}{\partial p_2}$: comparative statics of “cross-price” p_2
  - >0: (gross) substitutes
  - <0: (gross) complements

Note: whether a good is normal or inferior depends on preferences. i.e., the same good can be normal for one consumer but inferior for another.

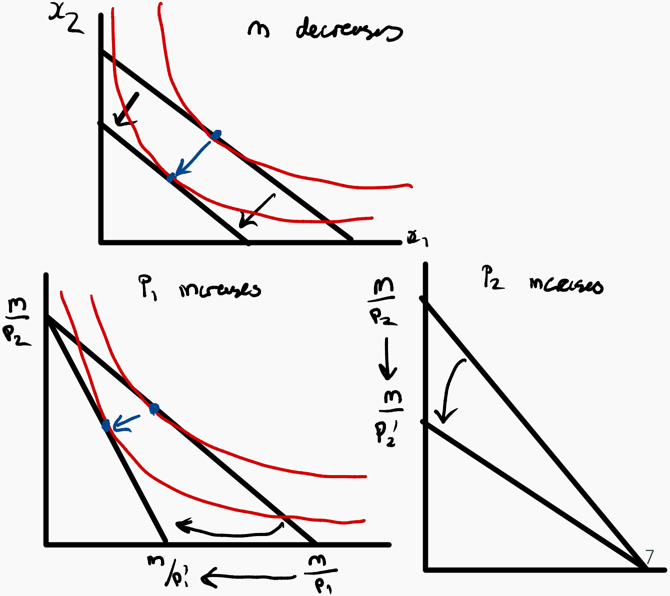
Or it may be normal for some levels of income and inferior in others for the same preferences.

similar: two goods can be complements for one person and substitutes for someone else

point is it all depends on preferences

How does the optimal outcome change as we increase/decrease one parameter and keep everything else the same (i.e., static)

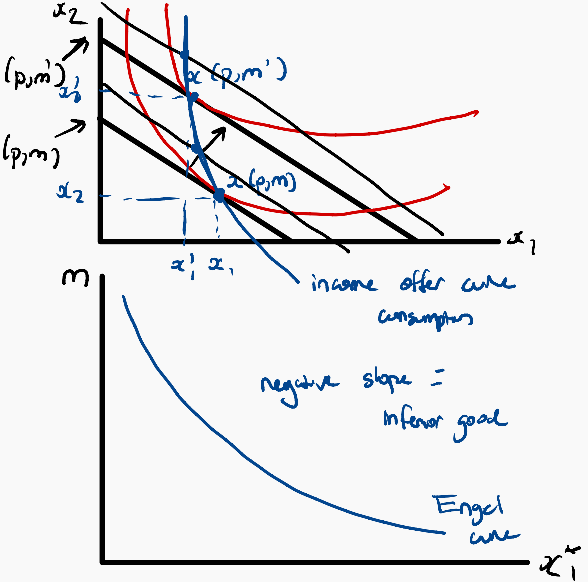
- Recall how our budget set's shape changes as we've changed these parameters/primitives
- Now we're combining this with what we know about the tangency condition and the different types of preferences
- Different preferences imply different effects on the solution



Comparative statics of income m

Changes in income m: budget increases outwardly in parallel

- **Normal goods**: goods you demand more of as income increases $x_2^* \uparrow$
  - Luxury goods
  - Necessary goods
- Inferior goods: goods you demand less of as income increases $x_1^* \downarrow$
- Graphical relationships:
  1. The ***income* offer curve**: traces how optimal bundle changes as income changes in $x_1 - x_2$ space
  2. The **Engel curve**: traces how optimal demand for a good changes in $x_1 - m$ space



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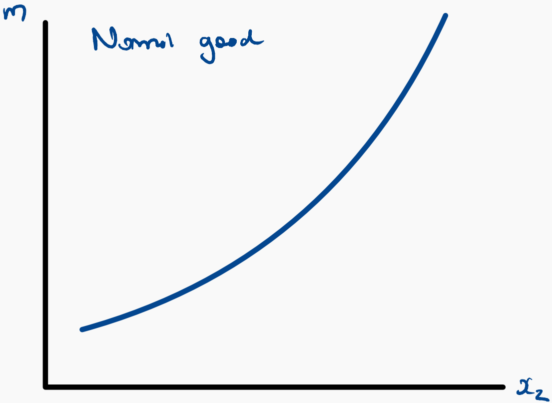
$$\frac{\partial x_2(p_1, p_2, m)}{\partial m} > 0$$

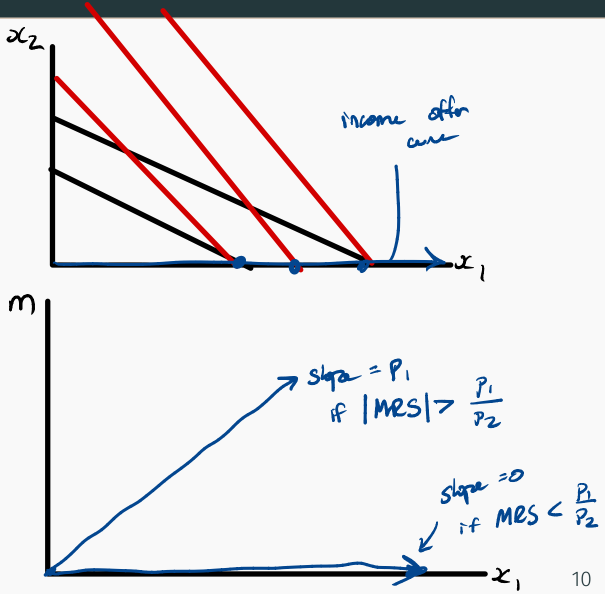


Changes in income m: budget increases outwardly in parallel

Perfect substitutes

$u(x_1, x_2) = \alpha x_1 + \beta x_2 + c$

- Previously found that $x_i^*(p_1, p_2, m) = \frac{m}{p_i}$ (proportional to income) or 0
- Income offer curve is the horizontal axis
- Engel curve is a straight line with slope p_1



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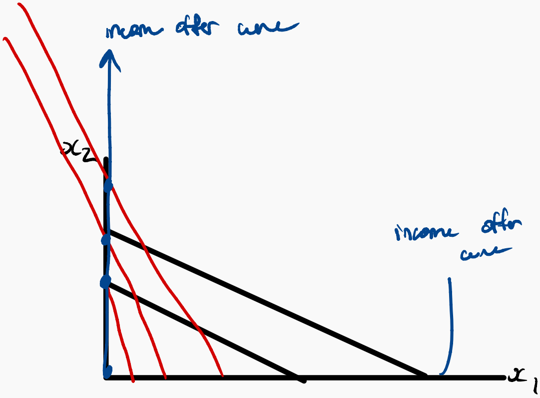


Changes in income m: budget increases outwardly in parallel

Perfect complements

$u(x_1, x_2) = \min\{\alpha x_1, \beta x_2\}$ ← $\alpha x_1 = \beta x_2$

$\Rightarrow x_2 = \frac{\alpha}{\beta} x_1$

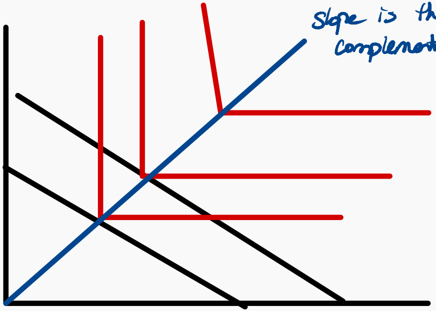
- Recall property that demand for good 1 and 2 always have the same form:

$$x_1^*(p_1, p_2, m) = \left(\frac{\beta \cancel{m}}{\beta p_1 + \alpha p_2}\right) m$$

$$x_2^*(p_1, p_2, m) = \left(\frac{\alpha \cancel{m}}{\beta p_1 + \alpha p_2}\right) m$$

- Income offer curve is a diagonal line through the origin
- Engel curve for x_1 is a straight line with slope $\frac{\beta p_1 + \alpha p_2}{\cancel{m}}$ ← β

Slope is the complementarity ratio $\frac{\alpha}{\beta}$



Engel curve slope $= \frac{\beta p_1 + \alpha p_2}{\beta}$

Changes in income m: budget increases outwardly in parallel

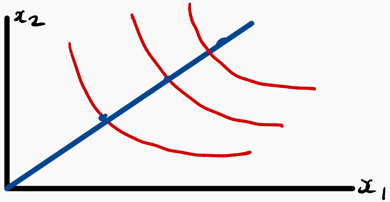
Cobb-Douglas $u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$ $\Longrightarrow$ $\lambda = \frac{\alpha}{\alpha+\beta}$

$u(x_1, x_2) = x_1^{\lambda} x_2^{1-\lambda}$

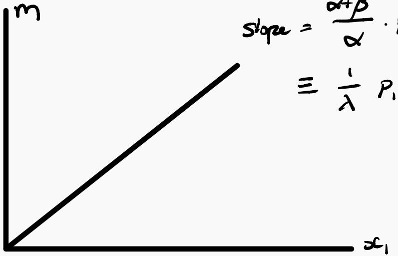
- Recall property that demand for good 1 and 2 always have the same form:

$$x_1^*(p_1, p_2, m) = \left(\frac{\alpha}{\alpha+\beta}\frac{\cancel{m}\,1}{p_1}\right) m \Rightarrow \frac{\lambda}{p_1} m$$

$$x_2^*(p_1, p_2, m) = \left(\frac{\beta}{\alpha+\beta}\frac{\cancel{m}}{p_2}\right) m \Rightarrow \frac{1-\lambda}{p_2} m$$



- Income offer curve is a straight line through the origin
- Engel curve is a straight line with slope $\frac{\alpha+\beta}{\alpha} p_1 \equiv \frac{1}{\lambda} p_1$



slope $= \frac{\alpha+\beta}{\alpha} \cdot p_1$

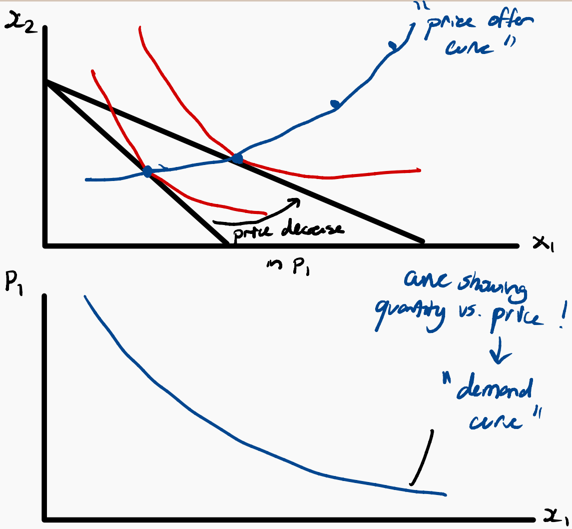
$\equiv \frac{1}{\lambda} p_1$

Comparative statics of price p

Changes in own price p_1: budget increases outwardly along one axis

Change in demand of **good 1** in response to a change in its own price p_1

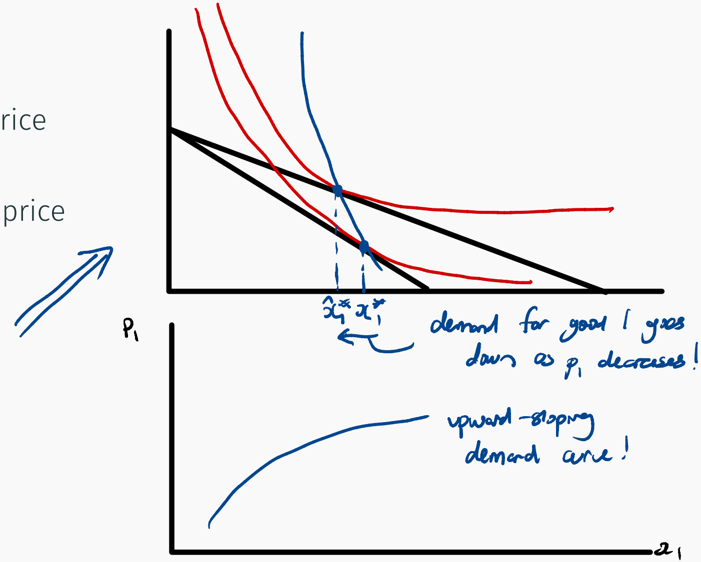
- *Price* **offer curve**: traces the bundles that would be demanded at different prices of good 1 in $x_1 - x_2$ space
- **Demand curve**: traces how optimal demand for a good changes in $x_1 - p_1$ space



Changes in own price p_1: budget increases outwardly along one axis

Change in demand of **good 1** in response to a change in its own price p_1

- Think of “law” of demand: as price increases, demand should decrease
- Exception to the “law”: Giffen goods
  - Theoretically possible
  - Pretty rare in reality



Changes in cross price p_1: budget increases outwardly along one axis

Change in demand of **good 1** in response to a change in **the other good's** price p_1

- (Gross) substitutes: $\frac{\partial x_1^*(p_1, p2, m)}{\partial p_2} < 0$
- (Gross) complements: $\frac{\partial x_1^*(p_1, p2, m)}{\partial p_2} > 0$

We've seen the extreme cases of

perfect substitutes: only demand good 1
or only demand good 2
depending on price ratio

perfect complements: only want the two goods
in the exact same fixed
proportions

More realistic: somewhere in between

Slutsky decomposition of the demand effects of a price change $(p \to p')$

Slutsky decomposition of the demand effects of a price change ($p \to p'$)

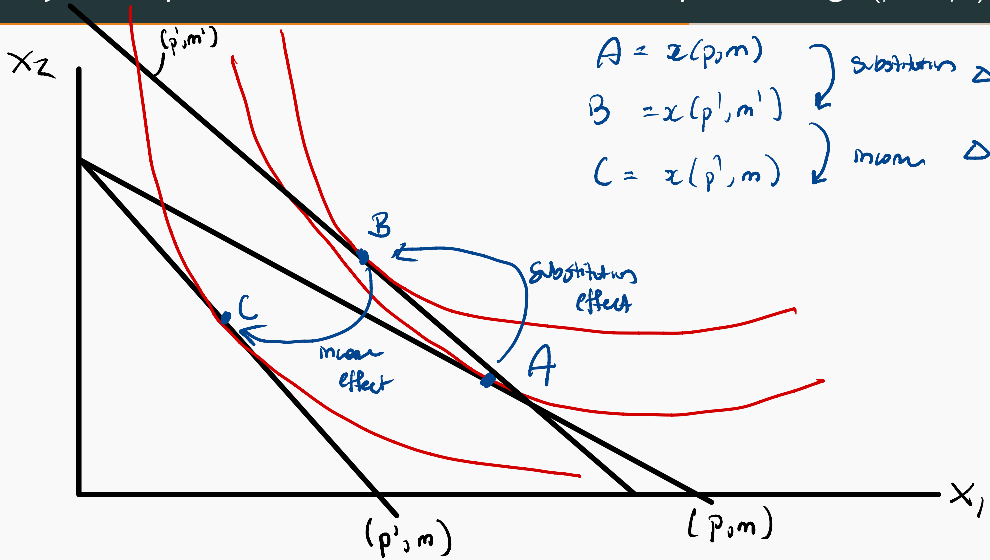
There are two reasons a price change would consumer's behavior:

1. Substitution effect: goods becoming more expensive incentivizes me to consume more of the other good (MRS vs. price ratio changes)
2. Income effect: goods becoming more expensive is in a sense equivalent to becoming poorer

The Slutsky decomposition tells us how big one effect is versus the other:

1. Holding fixed purchasing power, how much of the change in demand is due to changes in how the market values the two goods?
2. Holding fixed how the market values the two goods, how much of the change in demand is due to changes in purchasing power?

Slutsky decomposition of the demand effects of a price change ($p \to p'$)



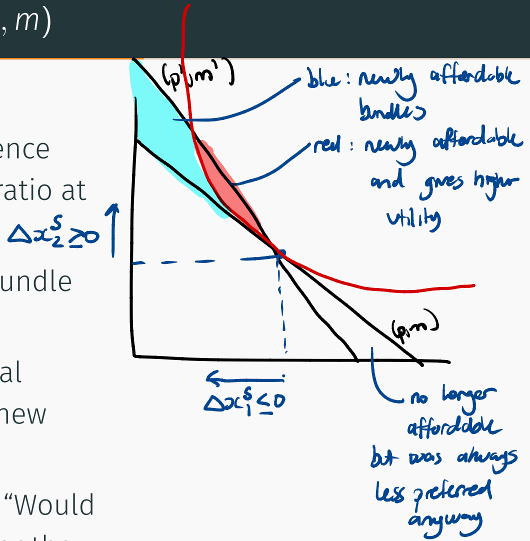
$A = x(p,m)$ $\left.\right\}$ Substitution Δx^s

$B = x(p', m')$ $\left.\right\}$ income Δx^n

$C = x(p', m)$

The substitution effect: $\Delta x^s = x(p', m') - x(p, m)$

- When $p \to p'$, the budget line changes slope
- MRS (slope of the highest attainable indifference curve) is in general no longer equal to price ratio at the same bundle
- Under the original budget line, the optimal bundle was just barely affordable
- But suppose we change income so the original bundle is again just barely affordable under new prices
- The substitution effect answers the question "Would the consumer demand the same bundle under the new prices if they could still afford it?"



The substitution effect: $\Delta x^s = x(p', m') - x(p, m)$

- When $p \to p'$, the original bundle is no longer "just" affordable. So change the budget from m to m' to compute this "compensated demand" that keeps purchasing power the same

Equation 1

$$
\begin{aligned}
m &= p_1 x_1 + p_2 x_2 \\
m' &= p_1' x_1 + p_2 x_2 \\
\Rightarrow \Delta m &= x_1 \Delta p_1
\end{aligned}
$$

$(x_1, x_2) = x^*(p, m)$

the optimal bundle under original prices

if price increases, I am positively compensated

If price decreases, I am negatively compensated to make up for my gain in purchasing power

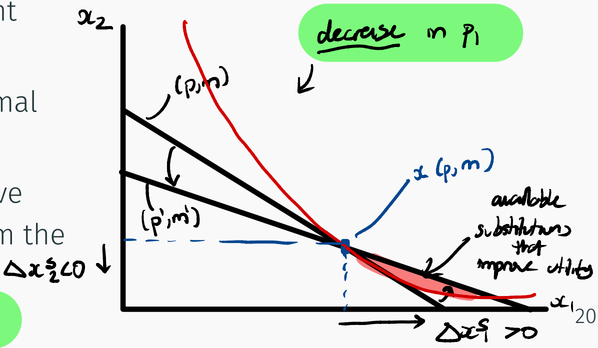
- Alternative framing: suppose I bought the bundle before the price change. I still have the same bundle after the price change. Does the new way the market prices the two goods allow me to trade off the bundle I bought for one that gives me higher utility?

The substitution effect: $\Delta x^s = x(p', m') - x(p, m)$

- Price ratio change means the market trades them off at different rates. This makes available some bundles that were previously unavailable.
- Under m', the original bundle is still just affordable, but the new price ratio might make a higher-utility bundle available
- The difference between these two optimal bundles is the substitution effect
- Substitution effect is always nonnegative since you wouldn't substitute away from the original bundle to a worse bundle

for a price decrease, nonpositive for a price increase!

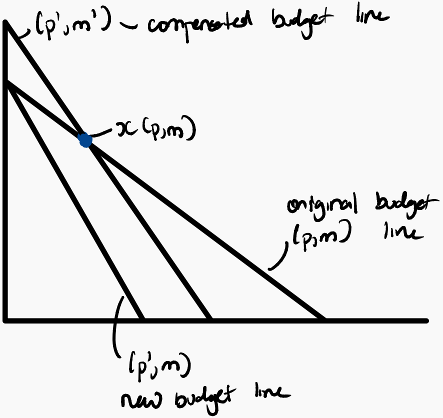
Can also think of it as "pivoting" the budget line around $x(p,m)$ until the price ratio is the new one: p'
This new budget line is (p', m'): the compensation is built in since $x(p,m)$ stays just barely affordable



The income effect: $\Delta x^n = x(p', m) - x(p', m')$

income effect

- Previous "compensated demand" case held fixed the affordability of original bundle, varied price $p \to p'$
- Now we want to hold prices fixed at new price p', and vary purchasing power $m \to m'$ to get an income effect
- Since we're comparing demand under different incomes m' and m, our discussion of normal/inferior goods and Engel curves becomes relevant
- Also means effect can be positive (if normal) or negative (if inferior).



Income effect compares demand under compensated budget line to new budget line, so holds prices at p', measures effect of m' vs. m

Total effect: $\Delta x = \Delta x^s + \Delta x^n$

$\hookrightarrow = x(p', m) - x(p, m)$

Effect of a price INCREASE

A → B : Δx^s substitution
B → C : Δx^n income
A → C : Δx total

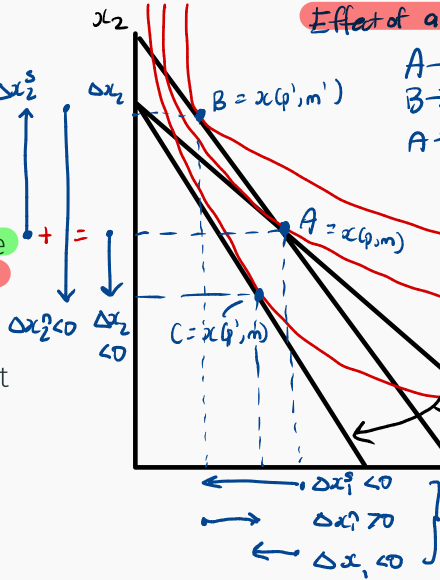
If the price ~~change~~ decrease is for a normal good, then income and substitution effects are both positive negative

If the price ~~change~~ decrease is for an inferior good, then income effect is negative positive and substitution positive negative

Giffen good: income effect is more negative than the substitution effect is positive (but needs to be very very very inferior)

FOR A PRICE DECREASE!

OPPOSITE SIGNS FOR PRICE INCREASE



∴ for these preferences good 1 is inferior!

General process for calculating the two effects

1. Using preferences, derive the general demand function $x^*(p, m)$
2. Solve for initial conditions (p, m), call this bundle A: $x^*(p, m)$
3. Given new price p', calculate the compensated income m' using Equation 1
4. Plug into the demand function to get bundle B: $x^*(p', m')$
5. Difference between bundles B and A is the substitution effect Δx^s
6. Plug new price and original budget into the demand function to get bundle C: $x^*(p', m)$
7. Difference between bundles C and B is the income effect Δx^n
8. The sum of the two effects is the total price effect we're familiar with:

$$\begin{aligned}\Delta x &= \Delta x^s + \Delta x^n \\ &= [x(p', m') - x(p, m)] + [x(p', m) - x(p', m')] \\ &= x(p', m) - x(p, m)\end{aligned}$$

Test your conceptual understanding

- Create the same graphs I drew for the case where p_1 *decreases* rather than increases
    - Label the three different budget lines according to the prices and budgets they correspond to
    - Label the set of bundles that become newly affordable/newly unaffordable
    - Label the substitution and income effects for both goods
    - Are they both positive/negative? Is your answer the same for both goods?
    - Indicate what conditions define a good as normal or inferior under that setting
- Do the same for when it's p_2 increasing and decreasing rather than p_1
- You'll find it's important how widely/curved you draw your budget lines and indifference curves makes graphically depicting the effects hard so practicing is important
- Pset 4, question 3d: how does Hicksian demand fit into this income/substitution effect framework?

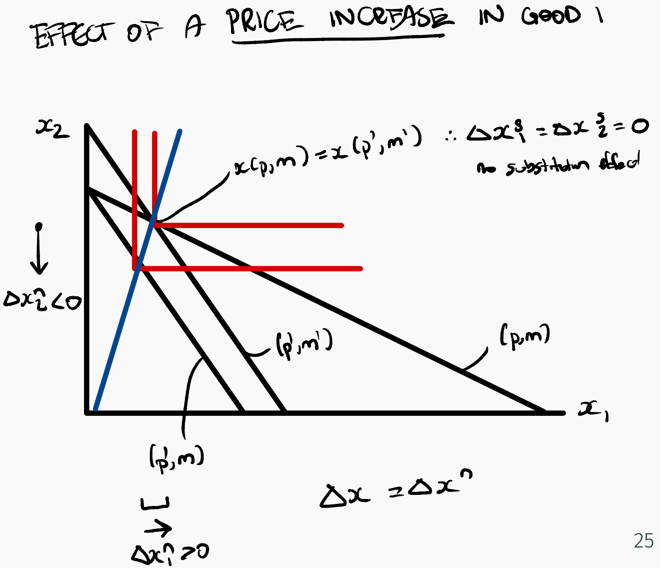
Slutsky decomposition of familiar preferences

Perfect complements

- Regardless of price ratio, original bundle will always be the desired bundle if it's just affordable:

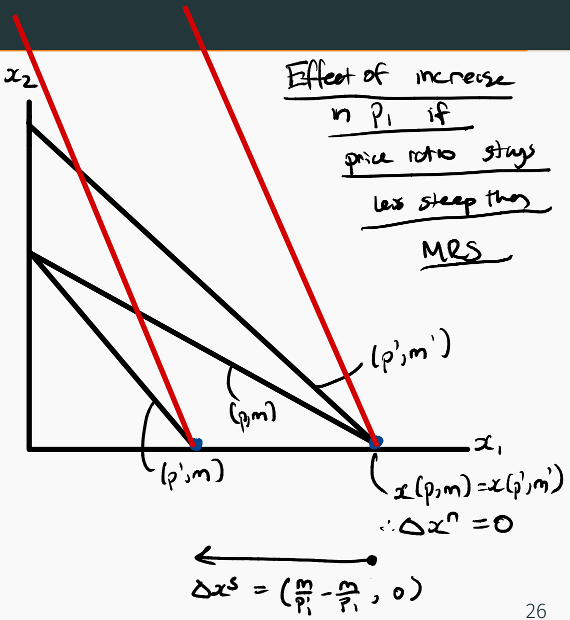
$$x(p', m') = x(p, m)$$

  for all p', keeping in mind that m' is a function of p' per Equation 1
- Thus there is no substitution effect
- All change in demand is driven by the income effect



Perfect substitutes

- If price changes enough in the right direction, then demand jumps from spending entirely on one good to spending entirely on the other
- The demand bundle jumps from the vertical axis (demand only good 2) to the horizontal axis (demand only good 1) or vice versa



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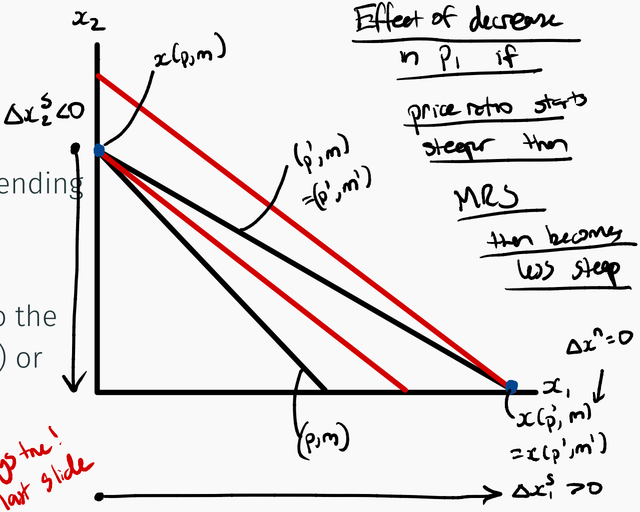


Effect of decrease in p_1 if price ratio starts steeper than MRS then becomes less steep

$$\Delta x^s = \left(\frac{m}{p_1'} , -\frac{m}{p_2} \right)$$

not always true! see last slide

But still $\Delta x^n = 0$: ~~always~~ no income effect for perfect substitutes

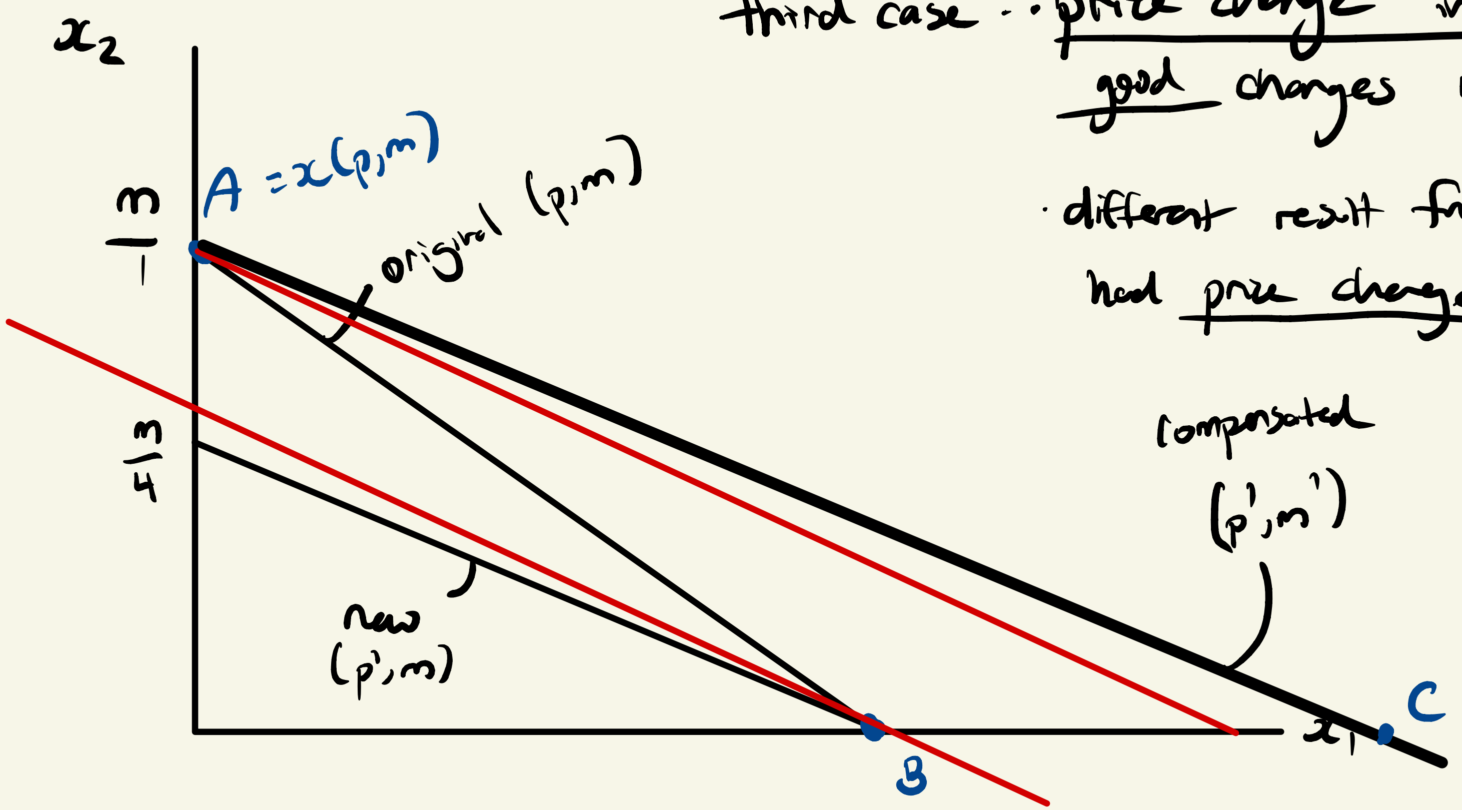
Perfect substitutes

- Thus, there is no income effect: the only thing that matters is the price ratio determining which good gives the highest marginal utility per dollar, which is a substitution effect

Third case: Perfect Substitutes + Non-zero Income Effect

third case: • price change in the initially demanded good changes which good is demanded

• different result from case two, which had price change in the initially not demanded good



A: $(0, +)$ $= x(p,m)$

B: $(+, 0)$ $= x(p',m)$

C: $(++, 0)$ $= x(p',m')$

substitution effect $(+, -)$

income effect $(+, 0)$